A Theory of Firm Boundaries with Long-Run Incentives and At-Will Employment*

Gorkem Bostanci — U of British Columbia
Ashwin Kambhampati — US Naval Academy

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Abstract: Existing theories of the firm define its boundaries through the optimal allocation of ownership rights of alienable capital. However, a broad set of business services today rely on inalienable human capital. We provide a new theory of firm boundaries based on dynamic incentives and at-will employment. In our model, in each period, a principal can either write a short-term outsourcing contract in which parties commit to ending the relationship after one period or an employment contract that potentially lasts multiple periods. In environments in which an employee's past success increases his cost of effort in future periods, the principal's lack of commitment to re-hire an employed worker undermines her ability to provide incentives in earlier periods. Hence, in equilibrium, there is too much worker turnover and too much outsourcing relative to the first-best assignment of workers to firms. We characterize when outsourcing contracts strictly outperform employment contracts, and vice-versa. Notably, our theory does not rely on capital ownership, heterogeneous adjustment costs, or pre-existing boundaries, cornerstones of existing theories of the firm.

Key Words: Firm Boundaries, Moral Hazard, Outsourcing

JEL codes: D23, D86, J41

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1 Introduction

The preeminent theories of firm boundaries rely on incomplete contracts and the ownership of alienable assets. Under contractual incompleteness, the Coase theorem does not hold and asset ownership matters for efficiency. Ownership provides non-contractible decision rights or access to non-contractible returns on the asset. Once the firm is defined as a collection of assets with common ownership over them, firm boundaries are determined by the efficient allocation of decision rights and non-contractual returns. These theories cohere well with classical make-or-buy decisions for physical inputs. But, 83 percent of intermediate inputs purchased in the U.S. economy in 2018 are services.[1] And, as Zingales (2000) and Rajan and Zingales (2001) have previously argued, it is hard to find an alienable asset whose ownership matters so much that it governs a manager's decision to hire versus outsource, say, a management consultant. [2]

This paper provides a novel theory of firm boundaries that does not rely on asset ownership. We posit a simple principal-agent model with at-will employment in which outsourcing provides a commitment to end the relationship after one period, whereas employment continues as long as both sides are willing. We show that if successfully completing a task today results in higher effort costs tomorrow, then the principal's inability to commit to retain an employed worker undermines her ability to provide incentives. Hence, outsourcing contracts may strictly outperform employment contracts, even when the latter are efficient.

Before describing the model and results in more detail, it is helpful to fix ideas with a concrete example. Consider the task of an internal auditor. When the auditor successfully completes his task, he potentially implicates his subordinates, coworkers, higher-ups, and, perhaps, himself. Hence, an employee assigned to complete the audit may choose not to be as thorough, considering how the audit impacts his long-term employment inside the firm. An external auditor, on the other hand, does not share the same

[1]See the Integrated Industry-Level Production Account (KLEMS) of the Bureau of Economic Analysis. Similarly, Atalay et al. (2014) report that about half of the upstream establishments report no shipments of tangible inputs to downstream establishments within the same firm.

[2]The distinguishing feature of the Professional, Scientific, and Technical Services subsector is the fact that most of the industries grouped in it have production processes that are almost wholly dependent on worker skills. In most of these industries, equipment and materials are not of major importance, unlike health care, for example, where "high tech" machines and materials are important collaborating inputs to labor skills in the production of health care. Thus, the establishments classified in this subsector sell expertise. https://www.census.gov/naics/2007NAICS/2007_Definition_File.pdf

concern because he commits to leaving the firm after the audit.

We study this type of incentive problem through the lens of a two-period principal-agent model in which, in each period, the principal chooses whether to outsource or write an employment contract. In an outsourcing contract, parties commit to a one-period relationship. The principal thus incurs an adjustment cost to find a new employee in period two. An employment relationship saves on adjustment costs, but an agent's successful completion of his task affects his cost of effort tomorrow, as in the auditor example. Moreover, at-will employment laws mean the principal can always replace the existing agent with a more attractive one in period two, i.e., the contracts the principal offers must be re-negation proof.

When successful task completion today lowers the expected cost of effort tomorrow, dynamic incentives of the principal and agent are aligned. However, when success today increases the expected cost of effort tomorrow, as in the auditor example, the agent has incentives to shirk today in order to reduce his chance of being replaced tomorrow. An outsourcing contract, through its commitment to limit the relationship to one period, eliminates these perverse incentives. Thus, the principal is more easily able to incentivize effort in the first period. As long as finding a new agent is not too costly, outsourcing thus outperforms an employment relationship.

Our analysis proceeds as follows. First, we characterize the first-best worker-task assignment (Proposition 1). We show that replacing an agent who fails in the first period is never efficient, while replacing an agent who succeeds in the first period is efficient if the cost of finding a new agent is smaller than the difference in the effort cost across the two periods.

We then characterize the principal's equilibrium choice of contract in terms of the model parameters (Proposition 2). The principal either (1) signs an employment contract in period one and always retains the worker, (2) signs an employment contract in period one and only retains the worker after low output, or (3) signs an outsourcing contract in period one. To decide whether to retain an employed worker, the principal compares the cost of finding a new agent to the cost of incentivizing effort in the second period. To decide whether to sign a "contingent employment" contract or an outsourcing contract, on the other hand, the principal compares the cost of finding a new agent to the dynamic rents the agent extracts in period one.

We observe that equilibrium contracting need not be efficient (Corollary 1) . In particular, the first-best worker-firm assignment is incompatible with outsourcing because outsourcing requires replacing an agent after failure. Yet, the principal chooses outsourcing for a non-trivial range of parameters of the model. Furthermore, there are parameters in which the equilibrium contract replaces an employed agent after success, while the first-best assignment retains them. In summary, the equilibrium contracting strategy often generates too much worker turnover and too much outsourcing relative to the efficient benchmark.

While our modeling is tailored to the auditor example, the general tension between dynamic incentives and commitment we identify appears relevant in a host of other problems. We suggest here two others. First, consider the task of internal capital allocation, a problem studied extensively in the literature on corporate finance.[3] A branch that receives a large share of resources today is likely to be more important to the firm tomorrow. Thus, a branch manager can be expected to overstate the resources needed in his branch for personal gains in the future. A management consultant, on the other hand, has better incentives because he expects little private gain from a growing branch. Second, consider an employer tasked with operating a new machine that might potentially be used to automate other tasks. The in-house operator may have incentives to misreport the machine's productivity — or sabotage the machine outright — if he or his co-workers face the risk of unemployment or reduced importance with successful automation. For example, in a retailer that is evaluating the performance of self-checkout machines, employed staff may be less willing to attend to problems generated by the machines or may overstate how frequently the machines malfunction.[4] An outsourced worker, on the other hand, would be less concerned about the future implications of automation if he expects to leave after performing his task.

1.1 Literature

This paper contributes to the strands of literature on firm boundaries and labor demand. We describe each and compare and contrast them with our model.

First, the Property Rights Theory (PRT) introduced in Grossman and Hart (1986) and

[3] See Stein (2003) for a review of the related literature.

[4] See Krueger and Mas (2004), Mas (2006), and Mas (2008) for evidence of explicit worker sabotage.

Hart and Moore (1990)[5] treats asset ownership as a tool to allocate residual decision rights for states of the world where contracts cannot be written or enforced. In PRT, asset ownership improves the outside option, thus, the surplus share of the owner. Hence, the asset owner faces a weaker hold-up problem and has better incentives to make ex-ante relation specific investments. The efficient distribution of productive assets optimizes the incentives to make relationship specific investments. The model prescribes hiring (make) when the firm's investments are more important and outsourcing (buy) when the agent's investments are more important. Importantly, ex-post decision making is efficient, even though it is not contractible. The inefficiency comes from the ex-ante investments. Similarly, in our model, the second period contracting is efficient although it is non-contractible and the inefficiency appears in the first period contract design. In contrast, in our model, it is the commitment power that makes outsourcing useful and not the residual control rights that follow capital ownership.

The second main theory of the firm concerns the appropriation of "quasi-rents" arising from integration versus non-integration, e.g., Williamson (1971) and Williamson (1975). We intentionally abstract from such considerations assuming, instead, that principals have all the bargaining power when contracting with either an in-house agent or an outsourced agent. In contemporaneous work, Raith (2021) formalizes a number of arguments in Williamson (1975) in order to provide answers to many of the same questions we ask.

The third main theory of firm boundaries is the Multitask Incentive Theory (MIT) developed in Holmstrom and Milgrom (1991) and Holmstrom and Milgrom (1994). In MIT, a principal (she) motivates a risk-averse agent (he) to allocate his effort across a multitude of tasks. When the effort can only be measured with noise, the principal needs to trade off providing high-powered incentives with minimizing the income risk of the agent. Asset ownership can be used as a tool to provide incentives. When keeping the asset valuable is important, the efficient incentive structure dictates that the agent owns the asset, while if other tasks are more important, the principal should own the asset. MIT thus uses ex-post non-contractibility to generate firm boundaries. Again, an alienable asset is at the center of the theory. Asset ownership is not important because it provides control over its use here, but because it gives the rights to the non-contractible returns on the assets. While we also build on a principal-agent structure, our setting does not include the main building blocks of MIT, i.e., effort allocation across tasks and capital maintenance.

[5] See Gibbons (2005) and Dessein (2014) for a broader review of the theoretical contributions.

The fourth main theory of firm boundaries builds on the literature on delegation (Stein (1997), Aghion and Tirole (1997) and Dessein (2002)). Similar to PRT, ownership provides the rights to make decisions on the use of the asset. Unlike PRT, these rights are exercised in equilibrium. The principal trades off making a decision herself versus delegating the decision to an agent who is better informed, but has perverse incentives and cannot credibly communicate his information to the principal.[6] While this trade off is silent on the firm boundaries at its core, it can speak to it if some decision rights can only be assigned to the asset owner. The efficient allocation of the assets optimizes the allocation of decision rights across agents.[7] Although we set our model up to be about effort provision, it can be reformulated as a model of communication and decision making. The dynamic moral hazard problem we highlight occurs when truthful communication today has negative effects on the agent tomorrow. In contrast to the literature, our model would define boundaries without capital ownership and the associated decision rights.

Finally, we contribute to the theoretical literature on dynamic contracts and the theory of the firm. Much of the literature on dynamic contracts with commitment focuses on facilitating inter-temporal risk-sharing (Rogerson (1985) and Spear and Srivastava (1987)). In contrast, in our setting, there are no risk-sharing opportunities (both parties are risk-neutral). Ohlendorf and Schmitz (2012) consider a contracting setting similar to ours in which all parties are risk-neutral and show that, if the agent is protected by limited liability, then dynamic incentives can relax single-period wealth constraints. In our setting, re-negation opportunities for the principal hamper commitment and prevent such possibilities.[8] Our paper thus shares similarities to those on relational contracting (e.g., Baker et al. (2002)) and those on the ratchet effect (e.g., Gerardi and Maestri (2020)). In our setting, in contrast to the relational contracting literature that concerns the theory of the firm, ownership of physical assets does not play a role. Relative to the literature on the ratchet effect, we do not study the role of private information that the agent possesses about the production technology. Instead, we focus on the effects of the principal's endogenously determined re-negation opportunities both on the optimal provision of incentives and on the optimal choice of labor inputs.

[6] Alonso et al. (2008), among others, conceptualize the problem as a CEO contemplating delegating decision rights to a branch manager. Then, the problem can be posited as the trade-off between adaptation (to local conditions) and coordination (between firm branches).

[7] See Baker et al. (2006) as a formalization of the argument.

[8] In Appendix B, we show that short-term contracts can replicate the performance of any long-term contract in our setting (see Fudenberg et al. (1990) for general conditions under which short-term contracts suffice).

2 The Basic Contracting Framework

2.1 Environment

Setup. There is a risk-neutral principal who has one task to be completed in each of two periods, $t = 1, 2$. In each period, she either writes an employment contract or an outsourcing contract with a single agent. This agent is drawn from a large pool of risk neutral agents who are protected by limited liability. Identifying and hiring a new agent, rather than retaining a previously employed worker, entails a cost of $\phi > 0$.

In each period t, the agent can exert effort to produce output. Let $e_t \in \{0, 1\}$ be her effort, where 0 corresponds to shirking and 1 corresponds to working. Let $y_t \in \{\underline{y}, \overline{y}\} := Y$ be the output she produces, where $\underline{y}$ corresponds to failure and $\overline{y} > \underline{y}$ corresponds to success. Effort e_t results in success with probability p_{e_t}, where $p_1 > p_0$.

For a new agent, the cost of working is $\underline{c} > 0$ and the cost of shirking is zero. If an employed agent continues her employment in period 2, then the cost of effort in that period depends on her success or failure in the first period.[9] In particular, if $y_1 = \underline{y}$, then the cost of effort remains $\underline{c} > 0$ in period 2, while if $y_1 = \overline{y}$, then it is $\overline{c}$. Notice that $\overline{c}$ can either be larger or smaller than $\underline{c}$. If $\overline{c} > \underline{c}$, then past success increases future effort costs, while if $\overline{c} < \underline{c}$, then the opposite occurs and the production technology exhibits "learning-by-doing".

The preferences of all parties in the model are represented by the discounted sum of expected per-period payoffs. The ex-post payoff of an agent in period t is given by

$$w_t - c_t,$$

where $w_t \in \mathbb{R}_+$ is the dollar value of the transfer she receives and $c_t \in \{\underline{c}, \overline{c}\}$ is the cost of effort. The principal's ex-post payoff in period t is

$$y_t - w_t - \phi_t,$$

where $y_t \in \{\overline{y}, \underline{y}\}$ is the output produced, $w_t \in \mathbb{R}_+$ is the transfer to the agent, and $\phi_t \in$

[9] As the cost of effort depends on period 1's outcome, rather than the effort put forth by the agent, output is a sufficient statistic for effort in period 2.

$\{0, \phi\}$ is the search cost of hiring the agent (0 if re-hiring an employed worker and $\phi > 0$, otherwise). All parties have a discount factor of $\delta \in (0, 1)$.

Contracts. There are two types of contracts: employment contracts and outsourcing contracts. An employment contract has at-will termination. On the other hand, under an outsourcing contract, one (or both) side *commits* to end the relationship after the first period.

Regardless of the type of the contract signed, the principal chooses a spot contract in each period t, i.e., a function $w_t : Y \to \mathbb{R}_+$, where wages are restricted to be non-negative to respect agent limited liability. We assume, throughout, that $\overline{y} - \underline{y}$ is sufficiently large relative to the other parameters that the principal always desires to hire some worker in each period and implement work.

2.2 The First-Best Assignment

The first-best worker-task assignment that implements work trades off the cost of hiring a new worker, ϕ, and the additional effort the worker needs to undertake after a potential success, $\overline{c} - \underline{c}$.

Proposition 1. *In the efficient worker-task assignment, an employment relationship is developed in the first period and*

i. the worker is retained after failure;

ii. if $\phi \geq \overline{c} - \underline{c}$, then the worker is retained after success; and

iii. if $\phi \leq \overline{c} - \underline{c}$, then the worker is replaced after success.

We next characterize the second-best worker-task assignment when the principal cannot observe effort and cannot commit to re-hiring an employed worker. To do so, we first identify the optimal outsourcing contract and then the optimal employment contract.

2.3 The Optimal Outsourcing Contract

The optimal outsourcing contract, $w_o^* = (w_o^*(\underline{y}), w_o^*(\overline{y}))$, solves the following wage-minimization problem:

$$\min_{w_o(\underline{y}), w_o(\overline{y}) \in \mathbb{R}_+} \quad p_1 w_o(\overline{y}) + (1-p_1) w_o(\underline{y})$$

subject to

$$[IC_o] \quad p_1 w_o(\overline{y}) + (1-p_1) w_o(\underline{y}) - \underline{c} \geq p_0 w_o(\overline{y}) + (1-p_0) w_o(\underline{y}).$$

By standard arguments, at the optimal contract, IC_o binds and wages are given by

$$w_o^*(\overline{y}) = \frac{\underline{c}}{p_1 - p_0} \quad \text{and}$$
$$w_o^*(\underline{y}) = 0.$$

Hence, the agent's expected payoff is

$$U_o := \underline{c} \left(\frac{p_0}{p_1 - p_0} \right) > 0$$

and the principal's expected payoff is

$$\Pi_o := p_1 \left(\overline{y} - \frac{\underline{c}}{p_1 - p_0} \right) + (1-p_1)\underline{y}.$$

It will also be useful to define the agent's expected payoff under an optimal spot contract when her effort cost is $\overline{c}$:

$$U_{\overline{c}} := \overline{c} \left(\frac{p_0}{p_1 - p_0} \right) > 0.$$

2.4 The Optimal Employment Contract

We now identify the optimal employment contract, working backwards from period 2.

Period 2. If the same agent is employed for a second period after having produced $\underline{y}$ in period 1, or if a new agent is hired as an employee, then the cost of effort in period 2 is $\underline{c}$. Hence, the optimal spot contract is identical to the optimal outsourcing contract, yielding the principal a period 2 profit of

$$\Pi_e^2(\underline{y}) := \Pi_o.$$

If, on the other hand, the employed agent is hired in period 2 after having produced $\overline{y}$ in period 1, his cost of effort is $\overline{c}$. Optimal wages are thus given by

$$w_2^*(\overline{y}) = \frac{\overline{c}}{p_1 - p_0} \quad \text{and}$$
$$w_2^*(\underline{y}) = 0,$$

yielding the principal a period 2 profit of

$$\Pi_e^2(\overline{y}) := p_1\left(\overline{y} - \frac{\overline{c}}{p_1 - p_0}\right) + (1 - p_1)\underline{y}.$$

Notice that this expression is smaller than the profit from employing an outsourced agent, Π_o, whenever $\overline{c} > \underline{c}$ and larger when $\overline{c} < \underline{c}$.

Period 2 Contracting Decision. Since $\phi > 0$, the principal always retains the employed agent after she produces $\underline{y}$. On the other hand, the principal only retains the employed agent after she produces $\overline{y}$ if the cost of finding a new agent outweighs the gain from contracting with an agent with a lower cost of effort,

$$\phi \geq \Pi_e^2(\underline{y}) - \Pi_e^2(\overline{y}) = (\overline{c} - \underline{c})\left(\frac{p_1}{p_1 - p_0}\right).$$

Period 1. The agent takes into account the principal's period 2 hiring decision when deciding how much effort to exert in period 1. There are two cases to consider: (a) the principal retains the agent whether or not she produced $\underline{y}$ and (b) the principal fires the agent when she produces $\overline{y}$ and retains the agent when she produces $\underline{y}$.

In case (a), the dynamic incentive constraint ensuring that period 1 effort is optimal is

$$[IC_e] \quad p_1(w_1(\overline{y}) + \delta U_{\overline{c}}) + (1 - p_1)(w_1(\underline{y}) + \delta U_o) - c \geq p_0(w_1(\overline{y}) + \delta U_{\overline{c}}) + (1 - p_0)(w_1(\underline{y}) + \delta U_o).$$

At the optimal contract, IC_e binds and wages are given by

$$w_1^*(\overline{y}) = w_o^*(\overline{y}) - \delta\left(\frac{p_0}{p_1 - p_0}\right)(\overline{c} - \underline{c}) \quad \text{and}$$
$$w_1^*(\underline{y}) = 0.$$

The principal's first period profit is thus

$$p_1\left(\overline{y} - \left(\frac{\underline{c} - \delta p_0(\overline{c} - \underline{c})}{p_1 - p_0}\right)\right) + (1 - p_1)\underline{y}.$$

Notice that, if $\overline{c} > \underline{c}$, the principal need not reward the agent as much for high output as under spot contracting, since the agent receives a higher expected utility in period 2 conditional on success in period 1. On the other hand, if $\overline{c} < \underline{c}$, the agent requires more period 1 rent to account for her reduced rent conditional upon success in period 2.

In case (b), the optimal period 1 contract implementing effort must satisfy the dynamic incentive constraint

$$[IC_e] \quad p_1 w_1(\overline{y}) + (1 - p_1)(w_1(\underline{y}) + \delta U_o) - \underline{c} \geq p_0 w_1(\overline{y}) + (1 - p_0)(w_1(\underline{y}) + \delta U_o).$$

At the optimal contract, IC_e binds and wages are given by

$$w_1^*(\overline{y}) = w_o^*(\overline{y}) + \underbrace{\delta\left(\frac{p_0}{p_1 - p_0}\right)\underline{c}}_{\text{Dynamic Rent}} \quad \text{and}$$
$$w_1^*(\underline{y}) = 0.$$

Notice that the wage paid upon success is unambiguously larger than under an outsourcing contract. The principal's first period profit is thus

$$p_1\left(\overline{y} - \left(\frac{\underline{c} + \delta \underline{c} p_0}{p_1 - p_0}\right)\right) + (1 - p_1)\underline{y} < \Pi_o,$$

and does not depend on the value of $\overline{c}$.

The Optimal Employment Relationship. If $\phi \geq \Pi_e^2(\underline{y}) - \Pi_e^2(\overline{y})$, then the optimal employment contract lasts two periods; the agent is hired in period 2 no matter her output in period 1. However, if $\phi < \Pi_e^2(\underline{y}) - \Pi_e^2(\overline{y})$, then the agent is re-hired in period 2 only if her period 1 output is $\underline{y}$.

2.5 Result

The following Proposition fully characterizes the optimal employment relationship in terms of the primitives of the model.

Proposition 2. *The optimal employment relationship is characterized by the following properties.*

1. *(Long Term Employment is Optimal)*
   If
   $$\phi \geq (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right),$$
   then the principal optimally writes an employment contract in period 1 and re-hires the agent in period 2 whether or not he succeeds.

2. *(Outsourcing is Optimal)*
   If
   $$\phi < (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right) \quad \textit{and} \quad \phi < \underline{c} \left(\frac{p_1}{1 - p_1} \right) \left(\frac{p_0}{p_1 - p_0} \right),$$
   then the principal outsources an agent in period 1 and contracts with another agent in period 2.

3. *(Contingent Employment is Optimal)*
   If
   $$\phi < (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right) \quad \textit{and} \quad \phi \geq \underline{c} \left(\frac{p_1}{1 - p_1} \right) \left(\frac{p_0}{p_1 - p_0} \right),$$
   then the principal optimally writes an employment contract in period 1 and re-hires the agent in period 2 only if he fails in period 1. If he, instead, succeeds, the principal fires the agent in period 2 and contracts with another agent.

Proof. See Appendix A for all proofs. □

We make a number of observations. First, for tasks in which $\overline{c} < \underline{c}$, i.e., those for which initial success makes the agent more adept, signing an employment contract and retaining the worker is optimal. This is a reasonable assumption in many cases and is consistent with employment being the dominant worker contract for firms.[10] Second, for tasks in which $\overline{c}$ is sufficiently larger than $\underline{c}$, the contract choice in the first period depends on the relative size of the dynamic rent accrued to the agent in period 1 and the cost of signing a new contract in period 2. If dynamic rents are sufficiently large, then the firm finds it too costly to motivate a worker to perform well. Third, the strict advantage of the outsourcing contract comes from its commitment power. If the worker knows that the contract will end, then it becomes easier to implement high effort. Finally, we remark that if

$$\frac{\overline{c}}{\underline{c}} < 1 + \frac{p_0}{1-p_1},$$

then there is no value of ϕ for which contingent employment can arise as an optimal contracting strategy. Put differently, the principal either outsources in period 1 and hires a new worker in period 2, or retains an employed worker for two periods.

2.6 Efficiency of the Equilibrium Contract

Unlike in the problem of determining the efficient worker-task assignment, the principal considers the dynamic rents captured by the agent in an employment relationship when this agent knows she cannot commit to retain her. This leads to a wedge between the efficient worker-task assignment and the equilibrium contract, as summarized in Corollary 1.

Corollary 1. *The following properties hold in equilibrium.*

i. There is excess turnover relative to the first-best assignment:

(a) If

$$\overline{c} - \underline{c} \leq \phi \leq (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right),$$

then the principal replaces an employed worker that is successful in period one, even though it is efficient to retain him.

[10] In 2019, about 11% of the U.S. workers were employed by labor outsourcing providers (Bostanci, 2021).

(b) It is always efficient to retain a worker that fails in period one, but if

$$\phi \leq \min\left\{\underline{c}\left(\frac{p_1}{1-p_1}\right)\left(\frac{p_0}{p_1-p_0}\right), (\overline{c}-\underline{c})\left(\frac{p_1}{p_1-p_0}\right)\right\},$$

then the principal always replaces him in equilibrium.

ii. The principal outsources too much:

(a) If

$$\phi \leq \min\left\{\underline{c}\left(\frac{p_1}{1-p_1}\right)\left(\frac{p_0}{p_1-p_0}\right), (\overline{c}-\underline{c})\right\},$$

then the principal outsources in period one even though it is efficient to write an employment contract in period one and retain the worker when he fails.

(b) If

$$\overline{c}-\underline{c} \leq \phi \leq \min\left\{\underline{c}\left(\frac{p_1}{1-p_1}\right)\left(\frac{p_0}{p_1-p_0}\right), (\overline{c}-\underline{c})\left(\frac{p_1}{p_1-p_0}\right)\right\},$$

then the principal outsources in period one even though it is efficient to write an employment contract and retain the worker in period two no matter her success or failure in period one.

Figure 1 illustrates the parameter regions identified in Corollary 1 with a numerical example. The line $y_1 : \phi = \overline{c} - \underline{c}$ divides the parameter set into two parts: above it, retaining a successful worker is efficient (the union of regions A, D, and E), while below it, replacing him is efficient (the union of regions B and C). On the other hand, the line $y_2 : \phi = (\overline{c}-\underline{c})\left(\frac{p_1}{p_1-p_0}\right)$ creates two other parts: above it, the principal chooses long-term employment and retains the worker she hires in period 2 independently of his performance (region A), while, below it, she replaces him in period 2 (regions B, C, D, and E). Hence, in regions D and E, a successful worker is replaced even though it is efficient to retain him, as identified in (i.). The wedge between y_1 and y_2 arises because the transfer required to motivate the agent with high effort cost is larger than his disutility.

For $\phi < (\overline{c}-\underline{c})\left(\frac{p_1}{p_1-p_0}\right)$, the line $y_3 : \phi = \underline{c}\left(\frac{p_1}{1-p_1}\right)\left(\frac{p_0}{p_1-p_0}\right)$ divides the parameter set into two parts: the principal chooses outsourcing in regions B and E, and chooses contingent employment in C and D. Because outsourcing replaces a failed worker in the second period, which is never efficient, there is too much worker turnover in regions B

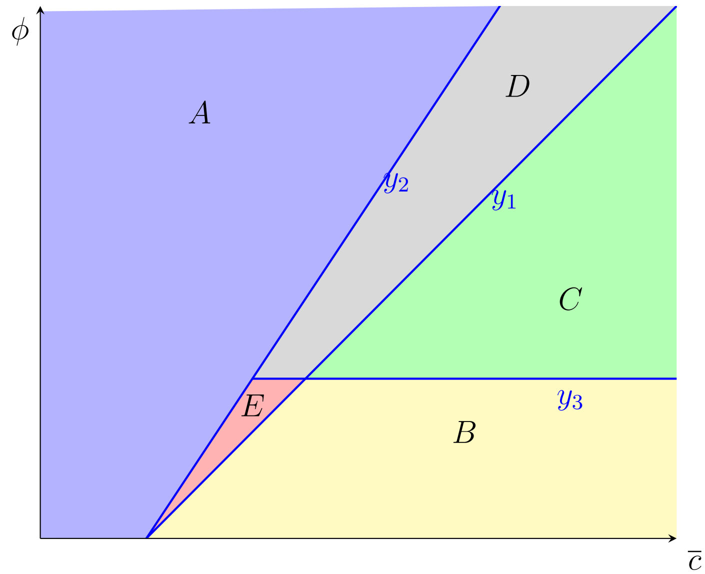

Figure 1: The Parameter Regions Characterized by the Efficiency and Equilibrium Conditions, $\underline{c} = 1, p_0 = 0.25, p_1 = 0.75$

and E. Even though a long-term contract would be efficient for region E and a contingent contract would be efficient for region B, the principal chooses to outsource as shown in (ii.). The discrepancy stems from the presence of dynamic rents: while the efficient assignment compares the increase in effort cost arising from hiring a successful worker with the cost of signing a new contract, the principal needs to take into account the additional compensation needed to implement high effort in the first period.

2.7 Discussion of Assumptions

Before embedding the contracting model into a full-fledged search-and-matching framework, we discuss two assumptions that we will maintain in the analysis going forward and how they might be justified.

Spot Contracting. In Appendix B, we prove that restricting attention to spot contracts is without loss of generality.

Commitment to Fire. In Appendix C, we show that outsourcing contracts involving a "commitment-to-fire" can arise endogenously without *apriori* restrictions on the set of feasible contracts.

Adjustment Costs. In Section 3, we endogenize adjustment costs through a labor market model characterized by search and matching. This extension allows for externalities in outsourcing decisions through labor market tightness.

3 A Two-Period Search Model

3.1 Environment

Matching. There is a measure one of principals and a measure $1 + \mu_a$ of agents, where $\mu_a \in (-1, \infty)$.

In each of two periods, $t = 1, 2$, each principal would like to hire an agent to complete a task. For simplicity, we assume that each principal is matched to an agent at the beginning of period 1. Let u denote the measure of unemployed workers at the start of period 2 and v denote the measure of vacancies at the start of period 2.

There is a matching function, $m : \mathbb{R}^2_+ \to \mathbb{R}_+$, that determines the measure of matches in period 2. We assume that it satisfies the following (standard) properties: (i) m is homogeneous of degree one; (ii) m is continuous and strictly increasing in both arguments; (iii) $m(0, v) = m(u, 0) = 0$; (iv) $m(u, v) \leq \min(u, v)$ with $m(u, v) < \min(u, v)$ when $u, v > 0$, so that there is frictional unemployment, and both $\lim_{u\to\infty} m(u, v) = v$ and $\lim_{v\to\infty} m(u, v) = u$. Hence, when $v > 0$, the probability with which a principal fills a vacancy in period 2, $q_v = m(u, v)/v = m(\frac{u}{v}, 1) \in [0, 1]$, depends only on the labor market tightness parameter $\tau := \frac{v}{u}$. When $v = 0$, $q_v = 1$ if $u > 0$ and $q_v = 0$ if $u = 0$ by convention. Similarly, when $u > 0$, the probability with which an unemployed worker finds a job in period 2, $q_u(u, v) = m(u, v)/u = m(1, \tau) \in [0, 1]$, depends only on the inverse of labor market tightness. When $u = 0$, $q_u = 1$ if $v > 0$ and $q_v = 0$ if $v = 0$ by convention.

Production Technology. If a principal and agent are matched, the agent can exert effort to produce output. Let $e_t \in \{0, 1\}$ be her effort, where 0 corresponds to shirking and 1 corresponds to working. Let $y_t \in \{\underline{y}, \overline{y}\} := Y$ be the output she produces, where $\underline{y}$ corresponds to failure and $\overline{y} > \underline{y}$ corresponds to success. Effort e_t results in success with probability p_{e_t}, where $p_1 > p_0$.

For an agent hired in period 1, the cost of working is $\underline{c} > 0$ and the cost of shirking is zero. If an agent continues her employment in period 2, then the cost of effort in that period depends on her success or failure in the first period.[11] In particular, if $y_1 = \underline{y}$, then the cost of effort remains $\underline{c} > 0$ in period 2, while if $y_1 = \overline{y}$, then it is $\overline{c}$. Notice that $\overline{c}$ can either be larger or smaller than $\underline{c}$. If $\overline{c} > \underline{c}$, then past success increases future effort costs, while if $\overline{c} < \underline{c}$, then the opposite occurs and the production technology exhibits "learning-by-doing". We will be interested in the cases in which $\overline{y} - \underline{y}$ is sufficiently large relative to $\overline{c}$ and $\underline{c}$ that the principal always desires to implement work.

Preferences. All parties are risk-neutral and their preferences are represented by the discounted sum of expected per-period payoffs. The ex-post payoff of an agent in period t is given by

$$w_t - c_t,$$

where $w_t \in \mathbb{R}_+$ is the dollar value of the transfer she receives and $c_t \in \{\underline{c}, \overline{c}\}$ is the cost of effort. The principal's ex-post payoff in period t is

$$y_t - w_t,$$

where $y_t \in \{\overline{y}, \underline{y}\}$ is the output produced, $w_t \in \mathbb{R}_+$ is the transfer to the agent. Any unmatched principal or agent receives a payoff of zero. All parties have a discount factor of $\delta \in (0, 1)$.

Contracts. Upon matching with an agent, a principal makes a take-it-or-leave-it contract offer. There are two types of contracts: employment contracts and outsourcing contracts. An employment contract has at-will termination. On the other hand, under an outsourcing contract, one (or both) side *commits* to end the relationship after the first period.

[11] As the cost of effort depends on period 1's outcome, rather than the effort put forth by the agent, output is a sufficient statistic for effort in period 2.

Regardless of the type of the contract signed, the principal chooses a spot contract in each period t, i.e., a function $w_t : Y \to \mathbb{R}_+$, where wages are restricted to be non-negative to respect agent limited liability.

3.2 Optimal Contracting in Period 2

Proceeding backwards, we characterize the optimal contracting strategy for the principals. In period 2, there is no distinction between an employment contract and an outsourcing contract (the game ends following this period). Given a cost of effort $\theta \in \{\underline{c}, \overline{c}\}$, the optimal contract, $w^*_\theta = (w^*_\theta(\underline{y}), w^*_\theta(\overline{y}))$, solves the following wage-minimization problem:

$$\min_{w_\theta(\underline{y}), w_\theta(\overline{y}) \in \mathbb{R}_+} \quad p_1 w_\theta(\overline{y}) + (1 - p_1) w_\theta(\underline{y})$$

subject to

$$[IC_2] \quad p_1 w_\theta(\overline{y}) + (1 - p_1) w_\theta(\underline{y}) - \theta \geq p_0 w_\theta(\overline{y}) + (1 - p_0) w_\theta(\underline{y}).$$

By standard arguments, at the optimal contract, IC_2 binds and wages are given by

$$w^*_\theta(\overline{y}) = \frac{\theta}{p_1 - p_0} \quad \text{and}$$
$$w^*_\theta(\underline{y}) = 0.$$

Hence, the agent's expected payoff is

$$u_\theta := \theta \left(\frac{p_0}{p_1 - p_0} \right) > 0$$

and the principal's expected payoff is

$$\pi_\theta := p_1 \left(\overline{y} - \frac{\theta}{p_1 - p_0} \right) + (1 - p_1) \underline{y}.$$

Notice that implementing effort is optimal in period 2 if and only if

$$p_1 \left(\overline{y} - \frac{\theta}{p_1 - p_0} \right) + (1 - p_1) \underline{y} \geq p_0 \overline{y} + (1 - p_0) \underline{y}$$

$$\iff \overline{y} - \underline{y} \geq \theta \left(\frac{p_1}{(p_1 - p_0)^2} \right) := \gamma_1(\theta, p_1, p_0).$$

3.3 Optimal Contracting in Period 1

In period 1, there are three contracts the principal might possibly write: (i) she can write an outsourcing contract; (ii) she can write an employment contract and retain the agent independently of their past success; and (iii) she can write an employment contract and retain the agent only if she fails. We discuss each in turn.

Outsourcing. Since the agent's continuation value is unaffected by her success or failure, the optimal outsourcing contract implementing effort is identical to the optimal period 2 contract under effort cost $\theta = \underline{c}$. The principal's present-discounted payoff is

$$\begin{aligned}\Pi_o =& p_1 \left(\overline{y} - w^*_{\underline{c}}(\overline{y})\right) + (1-p_1)\underline{y} + \delta q_v \pi_{\underline{c}} \\ =& (1+\delta q_v)\left(Y - p_1 \frac{\underline{c}}{p_1 - p_0}\right),\end{aligned}$$

where $Y \coloneqq p_1\overline{y} + (1-p_1)\underline{y}$. Implementing effort in period 1 is optimal whenever implementing effort in period 2 is optimal.

Unconditional Employment. Now, suppose that the principal retains the agent whether or not she succeeds. Suppose, also, that she offers a contract $(w_e(\overline{y}), w_e(\underline{y}))$, where $w_e(y)$ is the agent's wage upon producing output $y \in \{\underline{y}, \overline{y}\}$. By standard arguments it is optimal to set $w_e(\underline{y}) = 0$. Hence, under an optimal contract, the agent is willing to exert effort if and only if

$$\begin{aligned}& p_1 \left(w_e(\overline{y}) + \delta u_{\overline{c}}\right) + (1-p_1)\delta u_{\underline{c}} - \underline{c} \geq \\ & p_0 \left(w_e(\overline{y}) + \delta u_{\overline{c}}\right) + (1-p_0)\delta u_{\underline{c}}.\end{aligned}$$

At the optimal value of $w_e(\overline{y})$, the incentive constraint binds, yielding

$$\begin{aligned}w_e(\overline{y}) &= \frac{\underline{c}}{p_1 - p_0} - \delta\left(u_{\underline{c}} - u_{\overline{c}}\right) \\ &= \frac{\underline{c} - \delta p_0(\overline{c} - \underline{c})}{p_1 - p_0}.\end{aligned}$$

The principal's present-discounted payoff is

$$\begin{aligned}\Pi_e =& p_1\left(\overline{y} - w_e(\overline{y}) + \delta\pi_{\overline{c}}\right) + (1-p_1)\left(\underline{y} + \delta\pi_{\underline{c}}\right)\\ =& p_1\left(\overline{y} - w_e(\overline{y})\right) + (1-p_1)\underline{y} + \delta\left(p_1\pi_{\overline{c}} + (1-p_1)\pi_{\underline{c}}\right)\\ =& Y - p_1\left(\frac{\underline{c} - \delta p_0(\overline{c}-\underline{c})}{p_1-p_0}\right) + \delta\left(Y - \left(\frac{p_1\overline{c} + (1-p_1)\underline{c}}{p_1-p_0}\right)\right).\end{aligned}$$

Given that implementing effort in period 2 is optimal, implementing effort in period 1 is optimal whenever

$$(p_1-p_0)(\overline{y}-\underline{y}) - p_1\left(\frac{\underline{c}+\delta p_0(\overline{c}-\underline{c})}{p_1-p_0}\right) - \delta p_1(\overline{c}-\underline{c}) \geq 0$$

$$\iff \overline{y}-\underline{y} \geq \underline{c}\left(\frac{p_1}{(p_1-p_0)^2}\right) + \delta(\overline{c}-\underline{c})\left(\frac{p_1}{p_1-p_0}\right)^2 := \gamma_2(\overline{c},\underline{c},p_1,p_0,\delta).$$

Conditional Employment. Now, suppose that the principal retains the agent if and only if she fails. Suppose, also, that she offers a contract $(\hat{w}_e(\overline{y}), \hat{w}_e(\underline{y}))$, where $\hat{w}_e(y)$ is the agent's wage upon producing output $y \in \{\underline{y}, \overline{y}\}$. By standard arguments it is optimal to set $\hat{w}_e(\underline{y}) = 0$. Hence, under an optimal contract, the agent is willing to exert effort if and only if

$$\begin{aligned}& p_1\left(\hat{w}_e(\overline{y}) + \delta q_u u_{\underline{c}}\right) + (1-p_1)\delta u_{\underline{c}} - \underline{c} \geq\\ & p_0\left(\hat{w}_e(\overline{y}) + \delta q_u u_{\underline{c}}\right) + (1-p_0)\delta u_{\underline{c}}.\end{aligned}$$

At the optimal value of $\hat{w}_e(\overline{y})$, the incentive constraint binds, yielding

$$\begin{aligned}\hat{w}_e(\overline{y}) =& \frac{\underline{c}}{p_1-p_0} + \delta u_{\underline{c}}(1-q_u)\\ =& \frac{\underline{c} + \delta p_0(\underline{c} - \underline{c}q_u)}{p_1-p_0}.\end{aligned}$$

The principal's present-discounted payoff is

$$\begin{aligned}\hat{\Pi}_e =& p_1\left(\overline{y} - \hat{w}_e(\overline{y}) + \delta q_v\pi_{\underline{c}}\right) + (1-p_1)\left(\underline{y} + \delta\pi_{\underline{c}}\right)\\ =& p_1\left(\overline{y} - \hat{w}_e(\overline{y})\right) + (1-p_1)\underline{y} + \pi_{\underline{c}}\delta\left(p_1 q_v + (1-p_1)\right)\\ =& Y - p_1\frac{\underline{c}}{p_1-p_0}\left(1 + \delta p_0(1-q_u)\right) + \delta\left(p_1 q_v + (1-p_1)\right)\left(Y - p_1\frac{\underline{c}}{p_1-p_0}\right).\end{aligned}$$

Given that implementing effort in period 2 is optimal, implementing effort in period 1 is optimal if and only if

$$(p_1 - p_0)(\overline{y} - \underline{y}) - p_1 \hat{w}_e(\overline{y}) - \delta(p_1 - p_0)(1 - q_v)\pi_{\underline{c}} \geq 0$$

$$\iff \overline{y} - \underline{y} \geq \frac{p_1}{(p_1 - p_0)^2}\overline{c}(1 + \delta p_0(1 - q_u)) + \delta(1 - q_v)\left(p_1(\overline{y} - \frac{\underline{c}}{p_1 - p_0}) + (1 - p_1)\underline{y}\right).$$

This inequality satisfied for all values of q_u and q_v if

$$\overline{y} - \underline{y} \geq \left(\frac{1 + \delta p_0}{1 - \delta}\right)\left(\frac{p_1}{(p_1 - p_0)^2}\right)\overline{c} - \left(\frac{\delta}{1 - \delta}\right)\left(\frac{p_1}{p_1 - p_0}\right)\underline{c} := \gamma_3(\overline{c}, \underline{c}, p_1, p_0, \delta).$$

Feasibility of Unconditional Employment. The principal can commit to an unconditional employment contract, in other words, unconditional employment is *feasible* if and only if the principal cannot do better by firing the agent and filling the vacancy:

$$\pi_{\overline{c}} \geq q_v \pi_{\underline{c}} \iff q_v = m(\frac{1}{\tau}, 1) \leq \frac{Y - \frac{p_1 \overline{c}}{p_1 - p_0}}{Y - \frac{p_1 \underline{c}}{p_1 - p_0}}.$$

Similarly, conditional employment is feasible if and only if

$$\pi_{\overline{c}} \leq q_v \pi_{\underline{c}} \iff q_v = m(\frac{1}{\tau}, 1) \geq \frac{Y - \frac{p_1 \overline{c}}{p_1 - p_0}}{Y - \frac{p_1 \underline{c}}{p_1 - p_0}}.$$

Let $\hat{\tau}$ be the unique value at which both conditional and unconditional employment are feasible.

3.4 Equilibrium Characterization

To ensure that implementing effort is optimal in all scenarios, we maintain the following assumption in all our subsequent analysis.

Assumption 1.

$$\overline{y} - \underline{y} \geq \max\{\gamma_1(\overline{c}, p_1, p_0), \gamma_2(\overline{c}, \underline{c}, p_1, p_0, \delta), \gamma_3(\overline{c}, \underline{c}, p_1, p_0, \delta)\}.$$

Equilibrium in the case in which $\overline{c} < \underline{c}$ is completely characterized in the following proposition.

Proposition 3. *If $\overline{c} < \underline{c}$, then in the unique REE, every principal writes an unconditional employment contract.*

Proof. We first observe that only unconditional employment is feasible when $\overline{c} < \underline{c}$:

$$\frac{p_1\left(\overline{y} - \frac{\overline{c}}{p_1 - p_0}\right) + (1-p_1)\underline{y}}{p_1\left(\overline{y} - \frac{\underline{c}}{p_1 - p_0}\right) + (1-p_1)\underline{y}} > 1 \geq q_v.$$

In addition, unconditional employment strictly outperforms outsourcing because $u_{\overline{c}} > u_{\underline{c}}$ and $m(u,v) < \min(u,v)$ implies $q_v < 1$. $\square$

Proposition 3 suggests that with learning-by-doing, the optimal contract is simple. There is a larger surplus to retaining an employee, and with no commitment issues, unconditional employment becomes optimal. In the subsequent analysis, we restrict attention to the more interesting case where $\overline{c} > \underline{c}$. We make two preliminary observations.

Lemma 1. *If unconditional employment is feasible, then the principal obtains strictly higher profits from unconditional employment than conditional employment and outsourcing.*

The result in Lemma 1 is straightforward. If the unconditional employment is feasible, principal must find it optimal to retain the employee after success. If the principal finds it profitable to retain the employee ex-post, she must find it profitable ex-ante. The next lemma helps characterize the decision between outsourcing and conditional employment.

Lemma 2. *$\Pi_o \geq \hat{\Pi}_e$ if and only if $\tau \leq \overline{\tau}$, where $\overline{\tau}$ is the unique value of $\tau \in \mathbb{R}_+$ that solves the following equation:*

$$\frac{1 - m(\frac{1}{\tau}, 1)}{1 - m(1,\tau)} = \frac{\left(\frac{p_0}{1-p_1}\right)\left(\frac{p_1}{p_1 - p_0}\right)\underline{c}}{Y - \left(\frac{p_1}{p_1-p_0}\right)\underline{c}}. \tag{1}$$

Proposition 4. *The REE are characterized as follows.*

1. *If (and only if) $\mu_a \leq 0$, there exists a REE in which all principals credibly commit to unconditional employment contracts.*

2. *If* $\mu_a \geq \frac{1}{\overline{\tau}} - 1$*, there exists a REE in which all principals outsource. If* $\overline{\tau} > 1$*, the REE is unique for* $\mu_a > 0$*.*

3. *If* $\mu_a \leq p_1(\frac{1}{\overline{\tau}} - 1)$ *and* $\hat{\tau} < \overline{\tau}$*, there exists a REE in which all principals write conditional employment contracts.*

4. *If* $\mu_a \in (\min\{\frac{1}{\overline{\tau}} - 1, p_1(\frac{1}{\overline{\tau}} - 1)\}, \max\{\frac{1}{\overline{\tau}} - 1, p_1(\frac{1}{\overline{\tau}} - 1)\})$ *and* $\hat{\tau} < \overline{\tau}$*, there exists a REE in which a positive measure of principals write conditional employment contracts and a positive measure of principals outsource.*

Figure 2 demonstrates all REE as μ_a varies under $\overline{y} - \underline{y}$ sufficiently large, i.e., under $\overline{\tau} > 1$. There are a two important implications. First, as μ_a grows, the labor markets get less tight, and principals become less concerned about keeping the agent around. Hence, outsourcing becomes more prevalent in economies where the number of jobs is small relative to the size of the labor force.

Second, multiple types of contracts only co-exist for $\mu_a > 0$. Since the number of agents and principals is fixed, the unemployed and vacancies always change by the same magnitude. When $\mu_a < 0$, $v > u$ and an increase in u and v by the same amount decreases $\tau = \frac{v}{u}$. Thus, as more principals sign shorter-term contracts, labor market tightness decreases, and it becomes more profitable to sign shorter-term contracts. Hence, there is strategic complementarity to the contract choice. On the other hand, when $\mu_a > 0$, $v < u$ and an increase in u and v by the same amount increases $\tau = \frac{v}{u}$. Thus, as more principals sign shorter-term contracts, labor market tightness increases, making it less profitable to sign shorter-term contracts. Hence, there is strategic substitutability to the contract choice. Therefore, a mixed equilibrium only exists when $\mu_a > 0$.

Corollary 2. *Outsourcing becomes more prevalent when*

1. $\overline{y}$ *and* $\underline{y}$ *are smaller and*

2. $\underline{c}$ *is larger.*

Corollary 2 provides two comparative statics on the choice of outsourcing. First, outsourcing is more prevalent in economies where output values $\overline{y}$ and $\underline{y}$ are smaller. This is because outsourcing is associated with more worker turnover; hence, the principal's

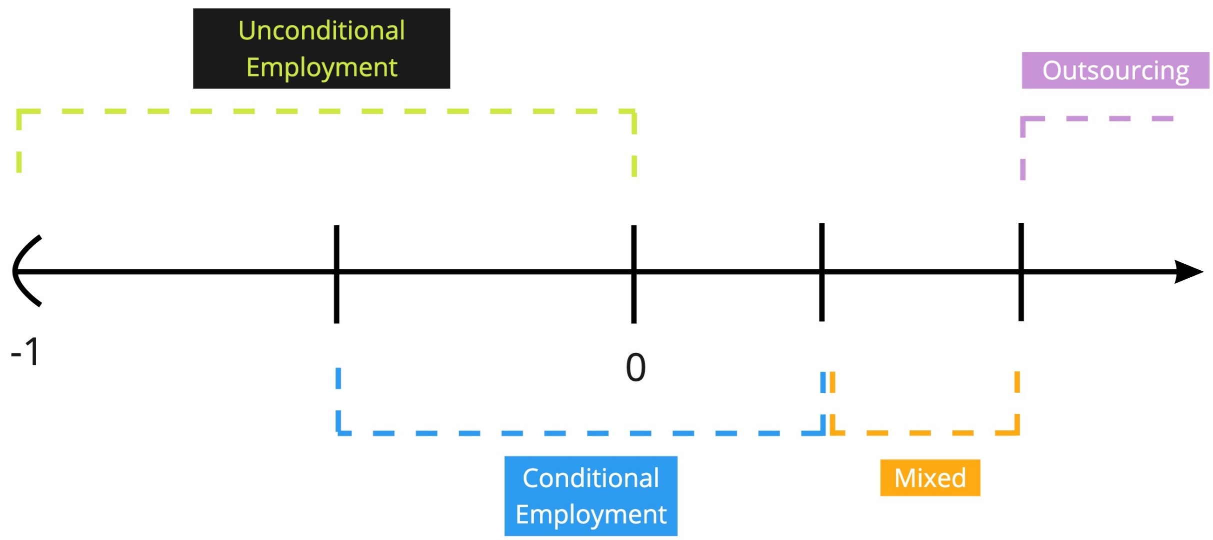

Figure 2: REE under $\overline{y} - \underline{y}$ large, with μ_a on the x-axis.

risk of remaining idle is higher. The size of the foregone profits of an idle principal determines how likely a principal is to sign an outsourcing contract. Second, outsourcing is more prevalent when the (low) effort cost $\underline{c}$ is larger in magnitude.[12] The additional compensation the principal needs to make to incentivize effort is proportional to $\underline{c}$; hence, it measures the intensity of dynamic incentive problems. As it grows, the principals are more likely to use outsourcing to avoid paying the associated rents to the agent.

4 Conclusion

In this paper, we set out to provide a theory of firm boundaries that fits the current landscape of labor outsourcing, where the outsourced service is provided using purely human capital. In our theory, boundaries are not defined by ownership of physical capital. Outsourcing contracts provide a commitment to end the relationship after one period. In contrast, employment continues as long as both sides are willing. We show that if completing a task today results in higher effort costs tomorrow, then the principal's inability to commit to retaining an employee undermines her ability to provide incentives. Hence, outsourcing contracts may strictly outperform employment contracts, even when the lat-

[12] The high effort cost ($\overline{c}$) is irrelevant because neither the conditional employment nor the outsourcing contract keeps an agent after success.

ter is efficient. Our results show that there is too much worker turnover in equilibrium relative to the efficient benchmark.

We are currently working on generalizing the current model in several dimensions, including extending it to an infinite horizon, allowing for continuous effort choice and output levels, and introducing alternative bargaining protocols between the principal and the agent. We are also formulating micro-foundations for changing effort costs consistent with the various examples we have provided in the Introduction.

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A Proofs

Proof of Proposition 1. For an employed agent, after failure in the first period, the cost of effort is identical to a new agent in the second period. Hence, the only difference is that bringing in a new agent has a cost of $\phi > 0$. Thus, keeping the same agent after failure creates strictly higher surplus. After success, an employed agent's cost of effort increases to $\overline{c}$. On the other hand, a new agent has only a cost of effort of $\underline{c}$. Hence, the efficient assignment replaces the old agent only if $\overline{c} - \underline{c} \geq \phi$. Hiring an employed agent in period 1 always generates strictly higher surplus than hiring an outsourced agent since the former potentially saves on search costs in period 2 with positive probability ($p_1 > 0$). □

Proof of Proposition 2. Recall that

$$\Pi_o = p_1 \left(\overline{y} - \frac{\underline{c}}{p_1 - p_0} \right) + (1 - p_1)\underline{y}$$

and

$$\Pi_e^2(\overline{y}) = p_1 \left(\overline{y} - \frac{\overline{c}}{p_1 - p_0} \right) + (1 - p_1)\underline{y},$$

and that the principal's expected payoff from a sequence of optimal outsourcing contracts is

$$\Pi_o + \delta(\Pi_o - \phi).$$

If $\phi \geq \Pi_o - \Pi_e^2(\overline{y}) = (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right)$, then the principal re-hires an employed agent whether or not she succeeds. Her payoff is thus

$$p_1 \left(\overline{y} - \left(\frac{\underline{c} - \delta p_0(\overline{c} - \underline{c})}{p_1 - p_0} \right) \right) + (1 - p_1)\underline{y} + \delta \left(p_1 \Pi_e^2(\overline{y}) + (1 - p_1)\Pi_o \right).$$

In period 1, the difference between profits from employment and outsourcing is

$$\delta p_0 \left(\frac{p_1}{p_1 - p_0} \right) (\overline{c} - \underline{c}).$$

In period 2, the difference between profits from employment and outsourcing is

$$-p_1(\Pi_0 - \Pi_e^2(\overline{y})) + \phi = -p_1^2 \left(\frac{\overline{c} - \underline{c}}{p_1 - p_0} \right) + \phi.$$

Employment thus outperforms outsourcing if and only if

$$\delta p_0 p_1 \left(\frac{\overline{c}-\underline{c}}{p_1-p_0}\right) - \delta p_1^2 \left(\frac{\overline{c}-\underline{c}}{p_1-p_0}\right) + \delta\phi \geq 0 \iff$$

$$\phi \geq (\overline{c}-\underline{c}) \left(\frac{p_1}{p_1-p_0}\right)(p_1-p_0) \, .$$

This condition always holds under the hypothesis that $\phi \geq (\overline{c}-\underline{c})\left(\frac{p_1}{p_1-p_0}\right)$ since $p_1 - p_0 \leq 1$.

On the other hand, if $\phi < \Pi_e^2(\underline{y}) - \Pi_e^2(\overline{y}) = p_1\left(\frac{\overline{c}-\underline{c}}{p_1-p_0}\right)$, then the principal fires the hired agent following success. Her payoff from the optimal employment contract is

$$p_1\left(\overline{y} - \underline{c}\left(\frac{1+\delta p_0}{p_1-p_0}\right)\right) + (1-p_1)\underline{y} + \delta\left(p_1(\Pi_o - \phi) + (1-p_1)\Pi_o\right) .$$

This yields the principal a higher payoff than a sequence of outsourcing contracts if and only if

$$\phi \geq \underline{c}\left(\frac{p_1}{1-p_1}\right)\left(\frac{p_0}{p_1-p_0}\right) .$$

□

Proof of Lemma 1. By Assumption 1, the principal optimally implements effort under all contracts. In addition, if $\overline{c} > \underline{c}$, then $w_e(\overline{y}) > \hat{w}_e(\overline{y})$ and $w_e(\overline{y}) > w^*_{\underline{c}}(\overline{y})$ so that expected wage payments are strictly smaller under unconditional employment than under conditional employment and outsourcing. It follows than her period 1 expected profits are strictly higher under unconditional employment. Finally, feasibility of unconditional employment implies that the principal obtains a weakly higher expected continuation utility in period 2 than under unconditional employment or outsourcing. The result follows. □

Proof of Lemma 2. Manipulating $\Pi_o \geq \hat{\Pi}_e$ yields

$$\underbrace{\delta p_1\left(\frac{p_0}{p_1-p_0}\right)\underline{c}(1-q_u)}_{\text{Period 1 Expected Wage Reduction Outsourcing}} \geq \underbrace{\pi_{\underline{c}}\delta(1-p_1)(1-q_v)}_{\text{Period 2 Expected Output Gain Conditional}} \iff$$

$$\frac{1-q_v}{1-q_u} \leq \left(\frac{p_1}{1-p_1}\right)\left(\frac{p_0}{p_1-p_0}\right)\left(\frac{\underline{c}}{\pi_{\underline{c}}}\right) \iff$$

$$\frac{1 - m(\frac{1}{\tau}, 1)}{1 - m(1, \tau)} \leq \left(\frac{p_0}{1 - p_1}\right)\left(\frac{p_1}{p_1 - p_0}\right)\left(\frac{\underline{c}}{\pi_{\underline{c}}}\right).$$

The right-hand side is strictly positive under Assumption 1. As the left-hand side is continuous in τ (by continuity of m in its arguments), approaches zero as τ approaches 0 (by $\lim_{u\to\infty} m(u, 1) = 1$ and $m(1, 0) = 0$) , and approaches infinity as τ approaches infinity (by $\lim_{v\to 0} m(1, v) = 1$ and $m(0, 1) = 0$), the intermediate value theorem guarantees the existence of a $\overline{\tau}$ that satisfies (1). As the left-hand side is strictly increasing in τ, there is only one such solution. In addition, $\Pi_o \geq \hat{\Pi}_e$ if and only if $\tau \leq \overline{\tau}$ as defined in the statement of the Lemma. □

Proof of Proposition 4.

1. If all principals choose an employment contract in period 1 and $\mu_a \leq 0$, then committing to retain the agent in period 2 is credible because $q_v = 0$. By Lemma 1, it is thus optimal for each principal to write an unconditional employment contract.

   For the only if direction, notice that if $\mu_a > 0$, then any principal can fire a successful worker and fill its vacancy with probability 1 if all other principals retain their workers. So,

   $$q_v = 1 > \frac{p_1\left(\overline{y} - \frac{\overline{c}}{p_1 - p_0}\right) + (1 - p_1)\underline{y}}{p_1\left(\overline{y} - \frac{\underline{c}}{p_1 - p_0}\right) + (1 - p_1)\underline{y}},$$

   where the inequality follows from $\overline{c} > \underline{c}$. Hence, it is not feasible to commit to unconditional employment.

2. If $\mu_a > 0$, unconditional employment is infeasible.[13] Any other REE must have a measure, $\alpha_c \in [0, 1]$, of principals writing conditional employment contracts and a measure, $\alpha_o \in [0, 1]$, of principals writing outsourcing contracts, with $\alpha_c + \alpha_o = 1$. In such an REE, the measure of vacancies in period 2 is

   $$v = \max\{0, -\mu_a\} + p_1\alpha_c + \alpha_o$$

[13]When unconditional employment is feasible, then it is strictly optimal to implement it. Hence, there cannot exist an equilibrium where only some of the principals sign an unconditional employment contract. Since we have shown that $\mu_a > 0$ rules out an equilibrium where all principals sign an unconditional employment contract, $\mu_a > 0$ must imply unconditional employment is infeasible.

and the measure of unemployed workers in period 2 is

$$u = \max\{0, \mu_a\} + p_1\alpha_c + \alpha_o.$$

Labor market tightness is thus

$$\tau = \frac{\max\{0, -\mu_a\} + p_1\alpha_c + \alpha_s}{\max\{0, \mu_a\} + p_1\alpha_c + \alpha_o}.$$

Notice that, if $\mu_a > 0$, then

$$\tau = \frac{p_1\alpha_c + \alpha_o}{\mu_a + p_1\alpha_c + \alpha_o}.$$

When $\alpha_o = 1$ (hence $\alpha_c = 0$), $\tau \leq \overline{\tau}$ if and only if $\mu_a \geq \frac{1}{\overline{\tau}} - 1$.

To show uniqueness under $\overline{\tau} > 1$, observe that $\tau < \overline{\tau}$ if and only if

$$\frac{p_1\alpha_c + \alpha_o}{\mu_a + p_1\alpha_c + \alpha_o} < \overline{\tau} \iff \mu_a > (p_1\alpha_c + \alpha_o)(\frac{1}{\overline{\tau}} - 1).$$

Then, with $\overline{\tau} > 1$ this equation holds for any $\mu_a > 0$. Hence, by Lemma 2, the only REE has all principals outsource.

3. Observe that if $\alpha_c = 1$ and $\alpha_o = 0$, then $\tau \geq \overline{\tau}$ if and only if $\mu_a \leq p_1(\frac{1}{\overline{\tau}} - 1)$. Hence, if $\hat{\tau} < \overline{\tau}$, so that unconditional employment is infeasible, an REE in which all principals write conditional employment contracts exists.

4. There exist interior values of α_c and α_o such that $\tau = \overline{\tau}$ if and only if $\mu_a \in (\min\{(\frac{1}{\overline{\tau}} - 1, p_1((\frac{1}{\overline{\tau}} - 1)\}, \max\{(\frac{1}{\overline{\tau}} - 1, p_1((\frac{1}{\overline{\tau}} - 1)\})$. If, in addition, $\hat{\tau} < \overline{\tau}$, then unconditional employment is infeasible. By Lemma 2, an interior equilibrium thus exists.

□

B Optimality of Spot Contracting

In Section 2, we assumed that the principal was constrained to use spot contracts. We prove here that this assumption is without loss of generality, i.e., that she cannot do better with a fully contingent contract. A fully contingent contract is a pair of functions (w_1, w_2), where $w_1 : Y \to \mathbb{R}_+$ and $w_2 : Y^2 \to \mathbb{R}$. $w_1(y_1)$ specifies wages in period 1 as a function of observed output in period 1, y_1. $w_2(y_1, y_2)$ specifies wages in period 2 as a function of observed output in period 1, y_1, and in period 2, y_2.

We first show that if the principal credibly retains the agent for two periods, then she cannot do better than use a sequence of spot contracts. Define

$$U_1 := p_1 w_1(\overline{y}) + (1-p_1) w_1(\underline{y}) - \underline{c},$$

$$U_2(\overline{y}) := p_1 w_2(\overline{y}, \overline{y}) + (1-p_1) w_2(\overline{y}, \underline{y}) - \overline{c}, \quad \text{and}$$

$$U_2(\underline{y}) := p_1 w_2(\underline{y}, \overline{y}) + (1-p_1) w_2(\underline{y}, \underline{y}) - \underline{c}.$$

The optimal dynamic contract that implements work in both periods and always (credibly) retains the agent solves

$$\min_{w_1, w_2} \quad U_1 + \delta(p_1 U_2(\overline{y}) + (1-p_1) U_2(\underline{y}))$$

subject to

$$[IC_1] \quad U_1 + \delta(p_1 U_2(\overline{y}) + (1-p_1) U_2(\underline{y})) \geq p_0 w_1(\overline{y}) + (1-p_0) w_1(\underline{y}) + \delta(p_0 U_2(\overline{y}) + (1-p_0) U_2(\underline{y}))$$

$$[IC_2(\overline{y})] \quad U_2(\overline{y}) \geq p_0 w_2(\overline{y}, \overline{y}) + (1-p_0) w_2(\overline{y}, \underline{y})$$

$$[IC_2(\underline{y})] \quad U_2(\underline{y}) \geq p_0 w_2(\underline{y}, \overline{y}) + (1-p_0) w_2(\underline{y}, \underline{y})$$

$$[R(\overline{y})] \quad p_1 w_2(\overline{y}, \overline{y}) + (1-p_1) w_2(\overline{y}, \underline{y}) \leq p_1 \frac{\underline{c}}{p_1 - p_0} + \phi$$

$$[R(\underline{y})] \quad p_1 w_2(\underline{y}, \overline{y}) + (1-p_1) w_2(\underline{y}, \underline{y}) \leq p_1 \frac{\underline{c}}{p_1 - p_0} + \phi.$$

Since only the differences $w(\overline{y}, y_2) - w(\underline{y}, y_2)$, $y_2 \in \{\underline{y}, \overline{y}\}$, matter for first-period incentives, it is without loss of generality to set $w(\overline{y}, \underline{y}) = w(\underline{y}, \underline{y}) = 0$. The second-period constraints thus simplify to

$$[IC_2(\overline{y})] \quad w(\overline{y}, \overline{y}) \geq \frac{\overline{c}}{p_1 - p_0}$$

$$[IC_2(\underline{y})] \quad w(\underline{y}, \overline{y}) \geq \frac{\underline{c}}{p_1 - p_0}$$

$$[R(\overline{y})] \quad w(\overline{y}, \overline{y}) \leq \frac{\underline{c}}{p_1 - p_0} + \frac{\phi}{p_1}$$

$$[R(\underline{y})] \quad w(\underline{y}, \overline{y}) \leq \frac{\underline{c}}{p_1 - p_0} + \frac{\phi}{p_1}.$$

From these constraints, we see that if

$$\phi < (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right),$$

then it is not possible to satisfy $IC_2(\overline{y})$ and $R(\overline{y})$ simultaneously, i.e. there is no incentive feasible contract that retains the agent in period 2 following success in period 1. If, on the other hand,

$$\phi \geq (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right),$$

then it is optimal to set $w(\underline{y}) = 0$. The first-period incentive constraint simplifies to

$$[IC_1] \quad w_1(\overline{y}) + \delta p_1(w_2(\overline{y}, \overline{y}) - w_2(\underline{y}, \overline{y})) \geq \frac{c}{p_1 - p_0} + \delta(\overline{c} - \underline{c}).$$

In light of this constraint and the principal's marginal rate of substitution between $w_2(\overline{y}, \overline{y}) - w_2(\underline{y}, \overline{y})$ and $w_1(\overline{y})$, she can do no better than use spot contracts, i.e. setting $w_2(\overline{y}, \overline{y}) - w_2(\underline{y}, \overline{y}) = 0$ and providing period 1 incentives solely through $w_1(\overline{y})$.

There are two other cases to consider. First, suppose the principal retains the agent only after failure. The optimal dynamic contract that implements work in both periods solves

$$\min_{w_1, w_2(\underline{y}, \overline{y}), w_2(\underline{y}, \underline{y})} \quad U_1 + \delta(1 - p_1)U_2(\underline{y})$$

subject to

$$[IC_1] \quad U_1 + \delta(1 - p_1)U_2(\underline{y}) \geq p_0 w_1(\overline{y}) + (1 - p_0)w_1(\underline{y}) + \delta(1 - p_0)U_2(\underline{y})$$

$$[IC_2(\underline{y})] \quad U_2(\underline{y}) \geq p_0 w_2(\underline{y}, \overline{y}) + (1 - p_0)w_2(\underline{y}, \underline{y})$$

$$[R(\underline{y})] \quad p_1 w_2(\underline{y}, \overline{y}) + (1 - p_1)w_2(\underline{y}, \underline{y}) \leq p_1 \frac{\underline{c}}{p_1 - p_0} + \phi.$$

In any solution to this program, it must be that $w_1(\underline{y}) = w_2(\underline{y}, \overline{y}) = 0$ (if not, then the principal could reduce wages by a small amount without affecting incentives and strictly increase her profits). The second-period constraints thus simplify to

$$[IC_2(\underline{y})] \quad w(\underline{y}, \overline{y}) \geq \frac{\underline{c}}{p_1 - p_0}$$

$$[R(\underline{y})] \quad w(\underline{y}, \overline{y}) \leq \frac{\underline{c}}{p_1 - p_0} + \frac{\phi}{p_1}.$$

On the other hand, the first-period constraint simplifies to

$$[IC_1] \quad p_1 w_1(\overline{y}) + \delta(1-p_1)p_1 w_2(\underline{y},\overline{y}) - \underline{c} \geq p_0 w_1(\overline{y}) + \delta(1-p_0)p_1 w_2(\underline{y},\overline{y}),$$

which holds if and only if

$$[IC_1] \quad w_1(\overline{y}) \geq \frac{\underline{c}}{p_1 - p_0} + \delta p_1 w_2(\underline{y},\overline{y}).$$

In light of $IC_2(\underline{y})$, it is thus optimal to set $w_2(\underline{y},\overline{y})$ as small as possible, i.e. equal to the optimal period 2 spot contract, so that $w_1(\overline{y})$ can be reduced by as much as possible. Hence, a sequence of spot contracts is optimal.

Second, suppose the principal retains the agent if she succeeds. The optimal dynamic contract that implements work in both periods solves

$$\min_{w_1, w_2(\overline{y},\overline{y}), w_2(\overline{y},\underline{y})} \quad U_1 + \delta p_1 U_2(\overline{y})$$

subject to

$$[IC_1] \quad U_1 + \delta p_1 U_2(\overline{y}) \geq p_0 w_1(\overline{y}) + (1-p_0) w_1(\underline{y}) + \delta p_0 U_2(\overline{y})$$

$$[IC_2(\overline{y})] \quad U_2(\overline{y}) \geq p_0 w_2(\overline{y},\overline{y}) + (1-p_0) w_2(\overline{y},\underline{y})$$

$$[R(\overline{y})] \quad p_1 w_2(\overline{y},\overline{y}) + (1-p_1) w_2(\overline{y},\underline{y}) \leq p_1 \frac{\underline{c}}{p_1 - p_0} + \phi.$$

In any solution to this program, it must be that $w_1(\underline{y}) = w_2(\overline{y},\underline{y}) = 0$ (if not, then the principal could reduce wages by a small amount without affecting incentives and strictly increase her profits). The second-period constraints thus simplify to

$$[IC_2(\overline{y})] \quad w(\underline{y},\overline{y}) \geq \frac{\overline{c}}{p_1 - p_0}$$

$$[R(\overline{y})] \quad w(\overline{y},\overline{y}) \leq \frac{\underline{c}}{p_1 - p_0} + \frac{\phi}{p_1}.$$

If

$$\phi < (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right),$$

then it is not possible to satisfy $IC_2(\overline{y})$ and $R(\overline{y})$ simultaneously. If, on the other hand,

$$\phi \geq (\overline{c} - \underline{c}) \left(\frac{p_1}{p_1 - p_0} \right),$$

then it is optimal to set $w(\underline{y}) = 0$. The first-period constraint simplifies to

$$[IC_1] \quad p_1 w_1(\overline{y}) + \delta p_1 p_1 w_2(\overline{y}, \overline{y}) - \underline{c} \geq p_0 w_1(\overline{y}) + \delta p_0 p_1 w_2(\overline{y}, \overline{y}),$$

which holds if and only if

$$[IC_1] \quad w_1(\overline{y}) + \delta p_1 w_2(\overline{y}, \overline{y}) \geq \frac{\underline{c}}{p_1 - p_0}.$$

In light of this constraint and the principal's marginal rate of substitution between $w(\overline{y}, \overline{y})$ and $w(\overline{y})$, she can do no better than setting $w_1(\overline{y}) = \frac{\underline{c}}{p_1 - p_0}$ and $w(\overline{y}, \overline{y}) = \frac{\overline{c}}{p_1 - p_0}$.

C An Example with Endogenous Commitment

Suppose now that there are three effort levels $\{e_l, e_m, e_h\}$ output levels $\{y_l, y_m, y_h\}$. In the first period, or after y_l, the cost of effort is $\{0, e_m, e_h\}$. The cost of effort becomes $\{0, \theta_m e_m, \theta_m e_h\}$ after y_m and $\{0, \theta_h e_m, \theta_h e_h\}$ after y_h. e_l generates y_m with p_0 and y_l with $1 - p_0$. e_m generates y_m with $p_1 > p_0$ and y_l with $1 - p_1$. e_h generates y_h with probability 1.

The idea is that it is efficient to implement e_m and keep the agent. However, the principal cannot commit to keeping the agent after e_m ex-post. The lack of commitment makes implementing e_m difficult as the agent will be inclined towards e_l. Hence, the principal may choose to implement y_h for certain tasks, and replace the agent with probability 1.

C.1 The Optimal Contract

Period 2. Suppose $y_1 = y_l$. Following the usual algebra, implementing e_m leads to

$$w_{e_m}^{y_l} = \frac{e_m}{p_1 - p_0}, \quad U_{e_m}^{y_l} = \frac{1 - p_1 + p_0}{p_1 - p_0} e_m, \quad \Pi_{e_m}^{y_l} = p_1 \left(y_m - \frac{e_m}{p_1 - p_0} \right) + (1 - p_1) y_l$$

Similarly, implementing e_h leads to

$$w_{e_h}^{y_l} = e_h, \quad U_{e_h}^{y_l} = 0, \quad \Pi_{e_h}^{y_l} = y_h - e_h,$$

and implementing e_l leads to

$$w_{e_l}^{y_l} = \frac{e_l}{1-p_0}, \quad U_{e_l}^{y_l} = 0, \quad \Pi_{e_l}^{y_l} = p_0 y_m + (1-p_0)(y_l - \frac{e_l}{1-p_0}).$$

The outcomes are similar for $y_1 = y_m$ and $y_1 = y_h$. We restrict attention to the parameter set where implementing e_m is optimal after y_l:

$$p_1 \left(y_m - \frac{e_m}{p_1 - p_0} \right) + (1-p_1) y_l \geq \max \left\{ y_h - e_h, p_0 y_m + (1-p_0) \left(y_l - \frac{e_l}{1-p_0} \right) \right\} \tag{2}$$

Notice that it is optimal to switch an agent with $y_1 = y_m$ when

$$\frac{p_1 e_m (\theta_m - 1)}{p_1 - p_0} > \phi \tag{3}$$

and it is optimal to switch an agent with $y_1 = y_h$ when

$$\frac{p_1 e_m (\theta_h - 1)}{p_1 - p_0} > \phi. \tag{4}$$

Period 1. In period 1, implementing e_m leads to

$$w_{e_m} = \frac{e_m(1+\delta p_0)}{p_1 - p_0}, \quad \Pi_{e_m} = p_1 \left(y_m - \frac{e_m(1+\delta p_0)}{p_1 - p_0} \right) + (1-p_1) y_l$$

Proposition 5. *The principal's contract strategy is fully characterized by the following properties.*

i. (Long Term Employment is Optimal)
If

$$\phi \geq e_m \frac{p_1 \theta_m - p_1 - p_0}{p_1 - p_0}$$

and

$$e_h + \delta p_1 \phi > \frac{e_m p_1}{p_1 - p_0} \left(\delta p_1(\theta_m - 1) + 1\right) + y_h - \left(p_1 y_m + (1 - p_1) y_l\right),$$

then the principal implements medium effort in period 1 and re-hires the agent in period 2 whether or not he succeeds.

ii. *(Contingent Employment is Optimal)*
If

$$\phi < e_m \frac{p_1 \theta_m - p_1 - p_0}{p_1 - p_0}$$

and

$$e_h \geq \left[y_h - (p_1 y_m + (1 - p_1) y_l) + \frac{e_m p_1 (1 + \delta p_0)}{p_1 - p_0}\right] \left(\frac{p_0}{p_1 - p_0}\right),$$

then the principal implements medium effort in period 1 and re-hires the agent in period 2 only if he fails in period 1. If he, instead, succeeds, the principal fires the agent in period 2 and contracts with another agent.

iii. *(Outsourcing is Optimal)*
If

$$e_h < \left[y_h - (p_1 y_m + (1 - p_1) y_l) + \frac{e_m p_1 (1 + \delta p_0)}{p_1 - p_0}\right] \left(\frac{p_0}{p_1 - p_0}\right)$$

and

$$e_h + \delta p_1 \phi < \frac{e_m p_1}{p_1 - p_0} \left(\delta p_1(\theta_m - 1) + 1\right) + y_h - \left(p_1 y_m + (1 - p_1) y_l\right),$$

then the principal implements high effort in period 1, fires the agent in period 2 and contracts with another agent.

Proof of Proposition 5.

$$\Pi_{LT} = (1 + \delta) \left[p_1 \left(y_m - \frac{e_m}{p_1 - p_0}\right) + (1 - p_1) y_l\right] - \frac{\delta p_1^2 e_m (\theta_m - 1)}{p_1 - p_0}$$

$$\Pi_{Cond} = (1 + \delta) \left[p_1 \left(y_m - \frac{e_m}{p_1 - p_0}\right) + (1 - p_1) y_l\right] - \delta p_1 \phi - \frac{\delta p_1 p_0 e_m}{p_1 - p_0}$$

$$\Pi_{Out} = y_h - e_h + \delta \left[p_1 \left(y_m - \frac{e_m}{p_1 - p_0}\right) + (1 - p_1) y_l\right] - \delta p_1 \phi$$

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