Negative Advertising and Competitive Product Positioning

Gorkem Bostanci Pinar Yildirim Kinshuk Jerath*

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Abstract

Negative advertising provides information about the weaknesses of a competitor's product. We study negative advertising with a focus on how its regulation impacts product positioning for profit-maximizing firms. We build a model of informative advertising competition, where product positioning is endogenous and consumers have rational expectations. We show that despite the informational benefits of negative advertising, permitting it (as the Federal Trade Commission in the United States does) may lead to reduced product differentiation and lower consumer welfare, even in markets where firms do not utilize negative advertising in equilibrium. We then extend our model to political competition, where a candidate's objective is to obtain a larger share of votes than the competitor. We show that political competition supports higher positional differentiation, along with more negative advertising than product competition, in line with observed high use of negative advertising in political races and their rarer use in product competition.

Key Words: *Negative Advertising, Product Positioning, Regulation, Political Campaigns*
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*Gorkem Bostanci is an Assistant Professor of Economics at University of British Columbia and a Visiting Assistant Professor at the University of Pennsylvania, Department of Economics. Pinar Yildirim is Assistant Professor at the Department of Marketing at the Wharton School. Kinshuk Jerath is Professor of Marketing at Columbia Business School. All errors are our own.

1 Introduction

Consider a firm entering a new market. One key strategic decision that the entrant has to make is how to position itself in the market. A major consideration in this decision is the positioning of the incumbents (Cooper and Kleinschmidt, 1987; Montoya-Weiss and Calantone, 1994). An entrant can position close to an incumbent or can carve out a position for itself by either differentiating its product design or choosing a brand image that differentiates it from the competitors (Fuchs and Diamantopoulos, 2010; Maarit Jalkala and Keränen, 2014). Advertising plays an important role for firms to inform consumers about product design or attributes (Meenaghan, 1995; Alden et al., 1999). The mention of an own product's strength or a competitor's weakness is known to help firms to emphasize the dimensions of differentiation to consumers (Grewal et al., 1997; Jewell and Saenger, 2014).

In response to the entrant, an incumbent may modify the design of its product (Carpenter, 1989; Ellickson et al., 2012; Seamans and Zhu, 2017), intensify positive advertising to remind consumers of its product, or tap into negative advertising to showcase the shortcomings of an entrant's product (Hauser and Shugan, 1983; Hauser and Gaskin, 1984; Kumar and Sudharshan, 1988). As modifications to product designs take longer, many firms focus on advertising as the first response strategy (Cubbin and Domberger, 1988; Thomas, 1999). Examples of incumbents utilizing negative advertising after a new entry are plenty. American Express faced abundant negative advertising from Visa and Mastercard while introducing its new card Optima, where the ads attacked its limited merchant coverage (Stevenson, 1988). The campaign was so effective that American Express downplayed its product introduction (e.g., offering it only to existing cardholders) to avoid further advertising war (Winters, 1987). After the deregulation of the Australian telephone industry, incumbent Telstra responded to the entry of Optus by running ads, emphasizing that Optus was not a domestic brand (Roberts, 2005). Similarly, entrants utilize negative advertising as a strategic tool accompanying their entry. The entry of Merck to the angiotensin-converting enzyme inhibitor market with Vasotec was accompanied by fierce negative advertising against Bristol-Myers Squibb's (BMS) Capoten. Merck ads emphasized Capoten's side effects, while BMS argued that studies could not confirm them. In the pain reliever market, McNeil's Tylenol faced two entrants, Datril, and Anacin, whose negative ads claimed inflammatory side effects from Tylenol (Knight, Knight; Robinson, 1988).

Negative advertising wars have been ubiquitous historically, although managers and advertisers complain that these wars harm all affected parties and decrease market demand altogether.[1] Beard (2013)(p.173), in his historical analysis on comparative advertising, provides a number of quotes from managers regarding the harm done by negative advertising. Bartos, a senior vice president for the agency J. Walter Thompson said regarding Coca-Cola's withdrawal from Cola Wars of the 1980s that "such strategies erode confidence in both brands in the mind of the public and that both companies would ultimately carry the soft-drink market into a commodity category" (Marketing News, 1980). McDonald's president Michael Quinlan is quoted saying regarding the Burger Wars "If you've got good [product], flaunt it, but don't tear down someone else. It's not good for the industry as a whole, and I think we ought to stop," (Hume, 1986). Following the Spaghetti Sauce Wars, an executive at Unilever proclaimed that "between [Unilever and Campbell], we're spending $60 million a year to convince consumers that our spaghetti sauce is really crappy" and "[during these wars] the category has declined every year for several years" (Neff, 1999).

There is sufficient evidence to show that industry leaders cannot avoid negative advertising wars, even though negative advertising harms all parties. Then, a rational entrant would make its product design decision considering the advertising competition down the road. As Robinson (1988) points out, "if aggressive and damaging reactions are expected, the entrant can be frightened off or choose to enter on a less ambitious scale" (p. 368). Evidence from political advertising markets shows that political candidates are less likely to use negative advertising when running against opponents who are ideologically similar to them (Ridout and Holland, 2010). Likewise, when little negative advertising is observed in an industry, this may be precisely because firms are making competitive product design choices to prevent it; and had negative advertising not been permitted, they would have made their design choices differently.

In this paper, we theoretically study how an entrant's product positioning is affected by the threat of downstream negative advertising response. To this end, we build a model of informative

[1] Beard (2010) documents how similar sentiments were also present back in the 1920s: "Writing about George Washington Hill's war on the sweets industry, the president of the New York Coffee and Sugar Exchange Inc., observed, "Has Mr. Hill forgotten that it was only a short time ago when some of our states, on health grounds, were legislating against cigarettes and that the term 'coffin nails' was applied to them? Would it not be well for the American Tobacco Company to 'Let sleeping dogs lie?' " (Lowry, 1929). A Printers' Ink author many years after the Baking Powder War warned that this kind of damage could last for years: "Lots of people still alive and well can vividly recall the days of some years ago when they were repeatedly warned to beware of 'benzoate of soda.' ... Eventually, various manufacturers and advertisers of foods discovered the alarming effect such copy was having on their business, and they recovered their reason by stopping all such publicity" (Hanley, 1927)." He substantiates the argument by adding that "In a speech to the Advertising Club of Greater Boston, David C. Stewart, president of agency Kenyon & Eckhardt, summarized this belief: "There are certain industries and certain product areas today in which the battle of competitive advertising claims has reached the harsh crescendo of jungle warfare ... public confidence [once] shaken ... [usually exerts] a stern reaction against the industries themselves" (as cited in Overly competitive ads invite action by U.S. 1965, 68)."

advertising competition where product design is endogenous. Each product in the market is characterized by three attributes: a horizontal design choice (i.e., position) that is explicitly chosen by the firm, and a negative and a positive vertical attribute, whose values are randomly drawn and revealed to firms after the product launch. The horizontal attribute of a product is directly observable by the consumer, and the value of the vertical attributes can only be revealed through firms' advertising. The design choice for the entrant is between two product positions that result in co-locating with or locating apart from the incumbent in a market where consumer preferences are horizontally differentiated. If the entrant co-locates, then it chooses a design similar to that of the incumbent, and competing products are more likely to have identical values for the positive and negative attributes.

As an example, which we will continue to refer to throughout the paper, consider two amusement parks that are in competition with each other due to the rides that they offer. Some characteristics of a ride can be differentiated along a horizontal dimension, for instance, whether it is for children or adults, which needs little advertising for consumers to learn. Other characteristics, such as the speed or safety of a ride, may vary along vertical dimensions and consumers may only learn about their values after being informed by ads.

In this environment, we model advertising as a firm's choice between truthfully informing consumers about the positive attributes of its own product ("positive advertising"), or truthfully informing consumers about the negative attributes of the competitor's product ("negative advertising"); or, not advertising at all. A firm's advertising choice also allows consumers to infer its unadvertised attribute(s). Furthermore, when products are similarly positioned in the market and their positive and negative attributes are positively correlated, one firm's advertising facilitates inference about the unadvertised attributes of the other product. Given the last point, firms may avoid highlighting the negatives of a competing product when their designs are close.[2] In the amusement park example, a park can focus on the speed of the rides through positive advertising, or the lack of safety features of a competing park's rides through negative advertising.

This simple structure, by itself, generates rich implications: (1) the presence or tone of advertising can be informative for both advertised and unadvertised products, and (2) the form of

[2]For instance, if McDonald's runs a negative advertisement about Burger King, criticizing the healthiness of Burger King products, consumers may be discouraged from eating at McDonald's as well, because McDonald's products are perceived similar to those of Burger King on a healthiness scale. The survey in Dolliver (2009) documents that 38% of consumers think less of the brand that does negative advertising. Beard (2013) writes about AnheuserBusch (A-B) executives admitting that their "[...] campaign criticizing craft brewers, such as the Boston Beer Company, for the questionable quality of their beer "garnered a stronger response from A-B consumers than from the non-A-B consumers they were targeting.""

advertising competition depends on how the products are positioned in the market. We endogenize the entrant's product positioning decision, which happens prior to the advertising decisions. A forward-looking entrant, therefore, considers potentially different advertising outcomes following each product positioning choice.

Our structure leads to a fundamental trade-off for the entrant. If the entrant chooses to position itself similarly to the incumbent, it cannot take advantage of the heterogeneous preferences of the consumer base. If it chooses a different product design than the incumbent, it opens itself to possible negative advertising by the incumbent. We analyze how this trade-off shapes the positioning and advertising strategies of firms.

Our first main result is that, in this setting, firms may practice negative advertising in a prisoners' dilemma outcome: if they could coordinate, both would prioritize positive advertising; however, competition pushes them to deviate to negative advertising. Furthermore, the incentive to deviate is stronger when their product positions are differentiated relative to when they are not. Our second main result is that firms have incentives to produce similar products at the product design stage to commit to not engaging in negative advertising later. If the benefit of avoiding a negative advertising war is larger than the cost of increased competition, firms choose designs that show higher similarity in equilibrium. Hence, negative advertising is more likely to be observed in markets where brands are sufficiently differentiated from each other in positioning. These findings are consistent with anecdotes from practice. In the drug industry, where side effects of medications are sufficiently salient, there is typically little product differentiation after patent expiration (Mandell and Hattem, 2019; Conti and Berndt, 2018). Similarly, in automotive industry where negative attributes such as crash ratings are often salient to drivers, there is limited differentiation (Thurk, 2018).

Given the potential for industry-level harm, it is natural to ask if negative advertising should be allowed. Despite the warnings from managers, the Federal Trade Commission (FTC) has adopted an encouraging position on negative advertising, claiming that mentioning a competitor "is a source of important information to consumers and assists them in making rational purchase decisions" and "encourages product improvement and innovation."[3] In contrast, the European Union had explicitly banned negative advertising until the late 1990s (Anderson and Renault, 2009). Given the opposing positions taken by the two regulatory agencies, it is essential that we investigate the welfare implications of negative advertising for consumers. Our structure is well-suited for such a

[3]See the FTC statement on comparative advertising here `https://www.ftc.gov/public-statements/1979/08/statement-policy-regarding-comparative-advertising`.

welfare comparison. In our model, negative advertising has two key welfare effects on consumers: first, as claimed by the FTC, consumers see a welfare gain due to "additional information" about the competing products. Second, counter to the FTC's claims, we show that negative advertising may incentivize "reduced product variety" in the market and lead to a welfare loss. Put differently; negative advertising can *discourage* product innovation. Our third main result is that, when prior uncertainty about the attributes of products is sufficiently low, the "reduced product variety" effect may dominate the "additional information" effect, and allowing negative advertising may result in an overall welfare loss for consumers. Furthermore, the welfare loss is higher when consumer preferences are more heterogeneous.

In the benchmark model, we abstract away from modeling price competition to deliver the intuition without added analytical complexity. In an extension, we examine the impact of endogenizing price competition in our setting, considering a model *a lá* Diamond (1971) and Kuksov (2004). Our fourth main result states that price competition reduces the incentives to co-locate to avoid negative advertising, but it does not entirely remove them as long as the heterogeneity in product preferences is sufficiently small.

In a second extension, we discuss a market where negative advertising is widespread — political competition. Our fifth main result is that negative advertising is more likely to be observed in political competition. This is because the objective of a politician is to win by plurality (i.e., receiving more votes than the competitor), rather than maximizing own vote count. This subtle modification to the objective function implies that a decrease in the overall voter base is not inherently bad for a politician; thus, the damage from running negative advertising is smaller for the candidates. We indeed observe that negative advertising is abundant in political competition (Ansolabehere et al., 1994; Gandhi et al., 2016). All of our results are robust to simultaneous rather than sequential entry.

Our paper contributes to three different strands of the literature. The first strand is on the theory of informative advertising competition. The literature starts with seminal papers by Grossman and Shapiro (1984), Austen-Smith (1987), and Meurer and Stahl II (1994) who analyze the role of information provision about product characteristics for competition in models where firms choose how much advertising to do.[4] These papers either do not explicitly consider consumers' inferences from advertising or assume consumers do not have rational expectations. Coate (2004)

[4] See LeBlanc (1998) and Amaldoss and He (2010) for examples of advertising competition where advertising informs consumers about prices instead of product attributes. See Anderson and Renault (2009) and Emons and Fluet (2012) for examples of "comparative advertising" models, where firms disclose information about both firms, in relation to one another.

introduces Bayesian voters who make inferences on unadvertised candidates, using the equilibrium advertising choices. Schultz (2007) introduces the ability to advertise about the opponent's type as well; however, a perfect unraveling result leads firms to advertise the type of both candidates or neither, thus prevents advertising tone from being a meaningful choice. The closest papers to ours are by Singh and Iyer (2020) and Polborn and David (2004). Both have a meaningful decision between positive and negative advertising in a Bayesian framework. We extend the framework of these papers in two directions, which are novel contributions to this literature, to the best of our knowledge. First, we model the correlation between competing product attributes, which allows advertising to have spillover effects beyond the advertised product. Second, we model product positioning and advertising tone decisions jointly as equilibrium outcomes. The two extensions allow our model to generate: (1) a prisoners' dilemma type negative advertising war, (2) reduced product variety by firms to avoid negative advertising wars, and (3) more candidate polarization in political competition relative to product competition.[5]

The second strand of the literature that we contribute to analyzes the role of negative advertising in political competition. The core mechanism that discourages firms from negative advertising in our case is the shrinking consumer base due to negative advertising. The empirical literature on political competition does not have a consensus on the effects of negative advertising on voter turnout.[6] One puzzle is why politicians utilize negative advertising, despite the ambiguous effect of the negative tone of advertising on their voter base. We contribute to this literature by demonstrating that, even in an environment in which negative advertising demobilizes one's own voters, political candidates may use more negative advertising relative to firms.

Finally, our paper is also related to the literature on product positioning. Gavish et al. (1983), Moorthy (1988), and Horsky and Nelson (1992) are among the large body of papers that study positioning decisions in the spirit of Hotelling (1929). Kuksov (2004) and Thomadsen (2007) show how the classical results in these models can be reversed once consumer search costs and asymmetric competitors, respectively, are taken into account. These papers study positioning in the absence of advertising decisions. We model the positioning decision jointly with advertising decisions and show

[5]See Skaperdas and Grofman (1995), Harrington Jr and Hess (1996), Bass et al. (2005), and Chen et al. (2009) for examples of persuasive, as opposed to informative, models of advertising competition through advertising tone. Bass et al. (2005) introduce brand and generic advertising, which are very close to our definitions of negative and positive advertising, respectively. They show that firms prefer generic advertising to enlarge the market in the short run and brand advertising to steal consumers from competitors in the long run. The model in Chen et al. (2009) yields a prisoners' dilemma similar to ours, however, through intensified price competition. Authors show that increasing the cost of advertising can help firms by preventing a pricing war that results in an advertising war.

[6]See Lau et al. (2007) and Arceneaux and Nickerson (2010) for no effect, Niven (2006) and Barton et al. (2016) for increased voter turnout, and Ansolabehere et al. (1994) for decreased voter turnout.

how the availability of negative advertising may lead to inefficiently low product differentiation.

The rest of the paper is organized as follows. In Section 2, we present the focal model in which competing entities have to make positioning and advertising decisions. In Section 3, we analyze the model for the case of firms selling products that have the objective of maximizing profits and obtain our key insights on positioning and advertising. In Section 4, we conduct a welfare analysis. In Section 5, we analyze the model for the case of political candidates aiming to win by plurality and obtain results on positioning and advertising that provide a contrast to the case of firm competition. In Section 6, we conclude. Analysis details and all proofs are provided in an online appendix to the paper.

2 Model

We build a model of informative advertising for competing but substitute products. We assume two competing products, each of which are defined by a horizontal attribute (product position/design) which is commonly known by everyone and two vertical attributes which can be disclosed through advertising. Consumers use information from advertising to infer the expected value of products before they make a purchase decision. We focus on the interdependence of product design and advertising by assuming that vertical product attributes are similar for similarly positioned products, which affects firms' positioning and advertising strategies. In this section, we explain how we model each of the above components in detail. We frame the main analysis focusing on product competition, and in Section 5.2 we discuss the implications for political competition.

2.1 Setup

Consumer Preferences There is a mass 1 of risk-neutral consumers whose ideal product is positioned at point L, and a mass 1 whose ideal product is positioned at point R. Consumers can either consume nothing for a utility of 0, or one of the available products in the market. The utility that consumer j at position $\chi_j \in \{L, R\}$ derives from consuming product i positioned at x_i is

$$U_{ij} = \gamma_j - |x_i - \chi_j| + A_i \tag{1}$$

where γ_j denotes the reservation value of consuming a product for consumer j, A_i represents the properties of product i that can only be revealed with advertising, and $|x_i - \chi_j|$ represents the positional distance between consumer j's ideal product (χ_j) and the position of product i

(x_i). We consider a horizontal differentiation model where consumers prefer products that are closer to their ideal product and incur a disutility from any deviation proportional to the distance between x_i and χ_j. The reservation value is heterogeneous across consumers according to the *cdf* $\gamma_j \sim \Gamma : [\underline{\gamma}, \overline{\gamma}] \to [0, 1]$. Throughout, we assume Γ is the uniform *cdf* for all derivations.

Firms and Products Two ex-ante identical firms (Firm 1 and Firm 2) produce substitute goods, and each firm chooses its product design and advertising strategy. The possible product designs are represented by the two consumer mass locations, L and R, where each position has an associated product design.

While consumers have zero distance to a product whose design is at their own location, the distance to the product at the opposite location is denoted by δ, which is a measure of heterogeneity in consumer tastes. A higher δ indicates that consumers are more dissimilar in their tastes, or the product in the opposite location has a lower match value. In this setting, positioning the two products at identical vs. differentiated locations can be interpreted as lower versus higher product variety in the market. We summarize the locations of the consumers and products in Figure 1.[7]

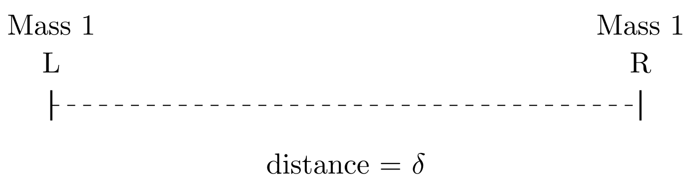


Figure 1: Consumer and Feasible Product Positions

The products in the market are also associated with two vertical attributes which are valued the same way by all consumers. For the exposition, we refer to the two vertical attributes as positive and negative, and denote them with P_i and N_i. We will refer to $\theta_i = \{P_i, N_i\}$ as a firm's type. There is uncertainty regarding the values of the positive and negative attributes, which we represent by using Bernoulli distributions:

$$P_i = \begin{cases} \Pi, & \text{w.p. } \sigma_\Pi \\ 0, & \text{o.w.} \end{cases} \quad \text{and } N_i = \begin{cases} -\beta, & \text{w.p. } \sigma_\beta \\ 0, & \text{o.w.} \end{cases} \tag{2}$$

[7] We can reach similar insights using a continuous distribution of consumers if Π is sufficiently larger than β; however, the change would make the derivations intractable. Previous studies have also used a discrete consumer distribution when product positioning is a decision (e.g., Kuksov, 2004). In Section 5.2, we consider a scenario where some consumers may have stronger preferences for one firm over the other and show that as long as the asymmetry is sufficiently small, the results continue to hold.

In the above formulation, $-\beta$ and Π indicate the valuation of the product attributes, while σ_β and σ_Π indicate the uncertainty around the valuation of products. The attributes P_i and N_i can be thought of as the outcomes of firms' experimentation when designing products. The realized values of these attributes to the consumers are unknown to the firms and to the consumers before the entrant makes the product location choice. Firms learn the attribute values for both products after they are designed, but before any advertising. The uncertainty in product attributes can be about the quality of each batch to be produced, safety issues that may be revealed over time, or external shocks that impact consumer tastes for existing attributes. Consumers can only learn the values of the vertical attributes through advertising. Throughout the paper, we use a convention where we say the positive (negative) attribute is "present" if $P_i = \Pi$ ($N_i = -\beta$). A_i in (1) then becomes the combined effect of the two attributes, i.e., the sum of N_i and P_i. We assume, without loss of generality, $\sigma_\Pi \Pi = \sigma_\beta \beta$, i.e., $E[A_i] = 0$.

The positioning choices of the products indicate an overlap between attributes such that, for products that are co-located, $cor(P_1, P_2) = cor(N_1, N_2) = \rho > 0$.[8] Hence, if product i has a negative attribute, i.e., $N_i = -\beta$, then the competitor's product is more likely to have the negative attribute as well, if its location is the same. We assume $cor(P_1, P_2) = cor(N_1, N_2) = 0$ when the products are located apart. There are also two implicit assumptions that are not necessary for the results, but simplify the exposition. First, regardless of their ideal position or reservation value, all consumers value the vertical attributes equally. Second, the values of the vertical attributes are identically distributed, regardless of the product location (L or R).

For the amusement park example, consider a ride designed for a particular age group. The positive attribute for this product can be its speed, whereas the negative attribute can be its safety risk. Naturally, products that are closer to each other along the horizontal attribute dimension (target age group) are more likely to have similar vertical attributes (speed and safety).

Advertising In this setting, advertising serves to inform consumers of the realized values of the product attributes. In particular, a firm can either advertise the presence of the positive attribute of its own product ($P_i = \Pi$) or the presence of the negative attribute ($N_i = -\beta$) of the

[8] In some industries such as pharmaceuticals, the ingredients used in drugs may be fairly common (especially when a patent expires), implying a high ρ between the competing drugs in the market. Therefore, the presence of a side effect in a branded drug may likely indicate the presence of similar side effects in generic drugs. In other markets where firms rely on trade secrets in the design of a product, such as in perfumery, ρ is expected to be low. A perfume having certain base notes tells little about what to expect from other perfumes that share similar top notes.

competing product, but not both.[9] We will refer to these choices as "positive" and "negative" advertising, respectively. Following up with the amusement park example, the speed of the rides can be highlighted through positive advertising, or the lack of safety features of a competing park's rides can be highlighted through negative advertising. Firms can also choose not to advertise; however, consumers make rational inferences from this choice. We assume that firms know the attributes of both products at the advertising stage and advertise truthfully.[10]

The firms and consumers have a common prior, which is identical to the distribution of products' attribute values given in (2). Consumers make rational inferences about the attributes through the advertising choices of the firms. Specifically, advertising can influence consumer valuation of a product positively or negatively. For example, if firm i announces $P_i = \Pi$, then consumers believe that the probability of $P_i = \Pi$ is 1, increasing their expected utility from product i. On the other hand, if firm $-i$ announces $N_i = -\beta$, then consumers will put probability 1 on $N_i = -\beta$, lowering their expected utility from purchasing product i.

Advertising a single vertical attribute, in our model, can have three distinct informative effects in equilibrium. First, it changes the consumer beliefs to a degenerate distribution about the advertised attribute. We refer to this as the "direct effect" of advertising. Second, it changes the consumer beliefs about the attribute(s) that are not advertised. Since advertising is a choice, the fact that a certain attribute is not advertised may inform consumers as well. We refer to this latter effect as the "inference effect" of advertising. Third, when products are co-located, and hence the realizations of their attributes are correlated, advertising may influence consumer beliefs about the competing product. We refer to this effect as the "spillover effect" of advertising.

The two firms' advertising may have rich combinations; however, our structure allows us to eliminate some. In particular, when all three effects are present for a given attribute, the "direct effect" always dominates the other two because the announcements are always truthful (as discussed in Footnote 10). The "inference effect," when present, dominates the "spillover effect" in any pure strategy equilibrium because, as will become clear shortly, information about the competing product

[9] This assumption is made for analytical tractability; either a budget constraint for firms or a limited attention assumption for consumers can rationalize one type of advertising. Moreover, similar to Polborn and David (2004), we do not allow advertising the absence of a vertical attribute.

[10] It is reasonable to question if the truth in advertising assumption that we are making is a reasonable one. Regulations in the United States and European Union (EU) both prevent firms from "untruthful" advertising – that is, if a firm is advertising that its product has characteristics that it does not have or provides benefits to consumers that it does not provide in reality, these ads are legally required to be removed. Therefore, even if firms could engage in untruthful ads in the short term, this cannot be a viable long-term strategy. Similarly, comparative advertising regulations in the EU require that a firm cannot claim a negative attribute for a competitor if the competitor does not really have this negative property. The guidelines of the Federal Trade Commission (FTC) of the US can be found at `https://www.ftc.gov/news-events/media-resources/truth-advertising`.

is (infinitely) more precise under the former effect relative to the latter. We will discuss these effects in detail in subsection "Inference from Advertising" in Section 3.2.

Consumers' and Firms' Problems Consumers choose between buying product 1, buying product 2, and not buying anything by comparing the expected utilities of each.[11] Consumers form their expectations using their posterior beliefs constructed from their common priors and the firms' advertising. The utility of the outside option is normalized to 0.

We consider a sequential entry scenario where one of the firms ("incumbent") is already located at L, and the other ("entrant") makes the location choice.[12] Without loss of generality, let Firm 1 be the incumbent firm and Firm 2 be the entrant and let the incumbent be located at the left end of the spectrum of consumer tastes (as provided in Figure 1), i.e., $x_1 = L$.[13] Entrant chooses its product positioning, $x_2 \in \{L, R\}$, and both firms choose advertising, $a_i \in \{P_i, N_{-i}, \varnothing\}$ to maximize revenues.

We assume that the market is competitive and firms are price takers, where the price is normalized to 1. This is done purposefully to keep our core model applicable to various cases of competition, such as product as well as political competition. This simplification makes it easier to comprehend advertising-related forces at play and how they interact with positioning. However, for the product competition case, we do include *price* as a decision variable in an extension of the model in Section 5.1. The main tension is that price competition is more intense with co-location than with differentiation; we build this tension into the model in Section 5.1 and show that our key results remain unchanged as long as price competition is not too intense.

Timeline of the Game The timing of decisions is given in Figure 2. First, Firm k, the entrant, chooses its product position to co-locate or differentiate itself from the position of the incumbent. Then, nature draws the values of the attributes $\theta = \{P_1, N_1, P_2, N_2\}$ for both products. At this stage, both firms observe θ, but consumers do not. Following this stage, the firms simultaneously choose their advertising — they decide whether to carry out positive advertising, negative advertising, or choose not to advertise. Finally, consumers receive advertising, update their beliefs about

[11] We assume that if two co-located products offer the same expected utility, consumers choose each good with equal probability. If two products that are located apart offer the same utility, however, we assume consumers choose the product that are located closer to their ideal location. The latter scenario only arises in parameter sets of measure zero, and our assumption simplifies the exposition.

[12] In the online appendix Section A.1, we also consider the case of simultaneous entry and show that the results are robust.

[13] Throughout the manuscript, we will use superscripts to denote policy functions and subscripts to denote realized decisions.

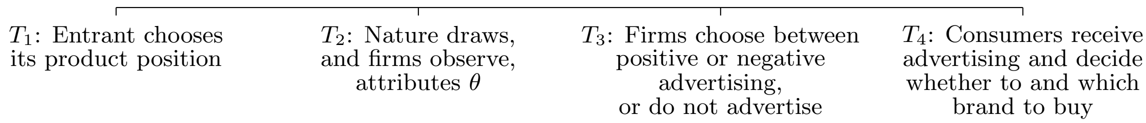


Figure 2: Timeline of the Incumbent-Entrant Game

the products, and make their product purchase decisions.

2.2 Equilibrium Definition

We will next characterize the equilibrium definition that we will utilize.

Definition Let $\theta_i = \{P_i, N_i\}$ denote the type of firm i and $\theta = \{\theta_1, \theta_2\}$. A Perfect Bayesian Equilibrium (PBE) in this game is a positioning decision for the entrant $x^2 \in \{L, R\}$, advertising decisions $a^2(x_2, \theta) \in \{P_2, N_1, \varnothing\}$, $a^1(x_2, \theta) \in \{P_1, N_2, \varnothing\}$ of firms, beliefs of consumers over firm types $\mathscr{F} : \theta_1 \times \theta_2 \to [0, 1]$, and purchase decisions of consumers $\{g^j(x_2, a_1, a_2)\}_j \in \{1, 2, \varnothing\}$ such that:

1. Consumer choices are sequentially rational, i.e., $\{g^j(.)\}_j$ maximizes $E[U_{ij}|\mathscr{F}]$.
2. $a^2(.)$ and $a^1(.)$ constitute a Nash Equilibrium (NE) of the advertising sub-game, given $\{g^j(.)\}_j$.
3. The location choice $x^2 \in \{L, R\}$ maximizes firm 2's profits, given $a^2(.)$, $a^1(.)$, and $\{g^j(.)\}_j$.
4. Consumer beliefs $\mathscr{F}$ are updated based on $a^2(.)$ and $a^1(.)$ according to the Bayes' Rule.

3 Product Positioning and Advertising Strategies

In this section, we characterize the solution to the model described in Section 2. To provide a benchmark, we first characterize the firm strategy in an environment where negative advertising is not allowed. Second, we characterize the solution to the full model, where both positive and negative advertising is permissible.

In the analysis below, we assume that there are always some consumers who will buy a product, and some who will buy neither. Assumption 1 ensures that trivial cases where advertising is ineffective are eliminated.

Assumption 1. $\underline{\gamma} < \Pi$ *and* $\overline{\gamma} > \delta + \sigma_\Pi \Pi + (1 - \sigma_\beta)\beta$

We will use backward induction to derive the equilibrium: first, characterize the equilibrium of the advertising sub-game, and then characterize the positioning strategy in the complete game.

3.1 Benchmark Model with Negative Advertising Not Permitted

We start the analysis with the consideration of a benchmark case where negative advertising is not permitted. For instance, until the 1990s, negative advertising was not permitted in numerous countries in the European Union (Anderson and Renault, 2009), leaving the only advertising options as running positive advertising or not running any advertising. Furthermore, since advertising is truthful, firm i can run positive advertising only if $P_i = \Pi$. Proposition 1 characterizes the equilibrium under this case.

Proposition 1. (Negative Advertising Not Permitted) *When negative advertising is not permitted, there exists a unique PBE in which the entrant locates its product apart from the incumbent, and only firms with positive attributes engage in advertising. Consumers are fully informed about positive attributes in this equilibrium, but not about the negative attributes.*

The proposition suggests that, when negative advertising is not permitted, firms advertise when they can, that is, when they have a positive attribute to announce ($P_i = \Pi$), and do not advertise otherwise. Consumers learn about the positive attribute of a product if the firm engages in positive advertising (direct effect of advertising) and infer that the positive attribute is missing (i.e., has a magnitude of 0) when it is not advertised (inference effect of advertising). Consumers cannot learn about the negative attributes of the products (N_i) since negative advertising is not allowed.

Recall that the common prior regarding the attributes ($P(P_i = \Pi) = \sigma_\Pi, P(N_i = \beta) = \sigma_\beta$) results in $E[A_i] = E[P_i + N_i] = \sigma_\Pi \Pi - \sigma_\beta \beta$, which we normalize to 0 by assuming $\sigma_\Pi \Pi = \sigma_\beta \beta$. On the one hand, if firm i runs positive advertising, consumers update their beliefs for product i positively (i.e., to $P(P_i = \Pi) = 1$), indicating that $E[A_i|a_i = P_i] = \Pi - \sigma_\beta \beta = (1 - \sigma_\Pi)\Pi$. On the other hand, running no advertising leads consumers to update their beliefs negatively (i.e., to $P(P_i = \Pi) = 0$), which results in $E[A_i|a_i = \varnothing] = 0 - \sigma_\beta \beta = -\sigma_\Pi \Pi$. Hence, regardless of the positioning of the products, the firms are weakly better off running positive advertising when they can. Lastly, the "spillover effect" is not present in this equilibrium because consumers can infer a product's positive attribute perfectly from the advertising decisions.

Next, let us consider the location choice of the entrant, which has to be made before the attribute values are realized. The entrant compares the expected payoff from choosing each location, where the expectation is taken over the potential realizations of the attributes of each product. Since negative advertising is not permitted, the realization of N_i becomes irrelevant.

When firm i has the positive attribute ($P_i = \Pi$), but the other firm is missing it ($P_{-i} = 0$), firm i will capture the entire market regardless of its positioning choice if $\Pi > \delta$, and capture the consumers who are at the same location with itself if $\delta \geq \Pi$. In the former case (when $\Pi > \delta$), the entrant's demand is unaffected by its location. In the latter case (when $\delta \geq \Pi$), the entrant would be better off locating apart if only the incumbent has the positive attribute. When firms are symmetric in attributes, i.e., $P_1 = P_2 = \Pi$ or $P_1 = P_2 = 0$ they share the market equally. If they are co-located ($x_2 = L$), then consumers located at R will have to do with a product that is not at their favorite position. If firms are located apart ($x_2 = R$), then all consumers have access to a product in their favorite position, and the aggregate demand will be larger, leading to a higher payoff for both firms. Hence, the expected payoff of locating apart is higher than that of co-locating, regardless of whether $\Pi > \delta$ or $\Pi \leq \delta$. Proposition 1 formalizes this reasoning.

When negative advertising is not permitted, consumers are not fully informed about product characteristics and may make decisions they regret ex-post. Proposition 1 suggests, however, that in the unique equilibrium, products are differentiated, which allows more consumers to buy a product matching their preferences. In the next subsection, we argue that allowing negative advertising leads to more informed consumers, yet may also reduce product differentiation in the market.

3.2 Full Model with Positive and Negative Advertising

Next, we analyze a model where negative and positive advertising are allowed; hence, the decision-set of the firm is to run positive ads, run negative ads, or not advertise. Notice that firm i can run positive advertising only if it has a positive attribute ($P_i = \Pi$) and run negative advertising only if its competitor has a negative attribute ($N_{-i} = -\beta$). We first define the concept of 'prioritized advertising' to simplify the exposition going forward.

Definition Firm i *prioritizes* positive (negative) advertising against firm $-i$ if it chooses to run positive (negative) over negative (positive) advertising when both are feasible, i.e., when the product of firm i has the positive attribute and the product of firm $-i$ has the negative attribute.

The trade-off for a firm between positive and negative advertising is as follows. Positive advertising by firm i increases the expected utility of consumers from consuming product i, and it hence may allow stealing consumers from its competitor, while expanding the market size at the same time. Negative advertising reduces the expected utility from consuming product $-i$, thus may allow stealing the competitor's consumers, but may shrink the overall market. Therefore, a firm would only find it attractive to engage in negative advertising if the gain from stealing customers exceeds the loss from shrinking market size. This trade-off can result in a *prisoners' dilemma* outcome in advertising: if they could choose, both firms would benefit from positive advertising. However, due to competition, they may find themselves in a negative advertising equilibrium, where they each experience losses due to the reduced market size. The presence of a "spillover effect" can help to prevent a prisoners' dilemma outcome by penalizing a firm for engaging in negative advertising when the competitor shares similar design features (i.e., has highly correlated attributes).[14] In anticipation of this spillover effect, the entrant may choose to co-locate to avoid an advertising war down the road. Proposition 2 formalizes this positioning strategy in anticipation of a subsequent advertising competition.

Proposition 2. (Co-location and Positive Advertising) *When negative advertising is permitted, there exists a PBE in which the entrant co-locates and both firms prioritize positive advertising. In the off-the-equilibrium path where the entrant locates apart, firms engage in negative advertising wars even when positive advertising is available.*

In Proposition 1 we showed that when negative advertising is not permitted, the unique equilibrium outcome is positive advertising while firms locate apart. Proposition 2 points to the possibility of another equilibrium with a counterintuitive outcome when negative advertising is allowed: The entrant may choose to co-locate with the incumbent to activate spillover effects and reduce the chances of a downstream negative advertising attack on itself. This strategy, in turn, pushes the incumbent and the entrant to engage in positive advertising. On the one hand, with no negative advertising, firms prevent the shrinkage of total market demand. On the other hand, co-location reduces the variety of products offered to consumers. Put differently; there is a direct relationship between advertising wars and product differentiation. While a firm would always prefer to differentiate its position for a given advertising outcome, if it chooses its advertising endogenously with the product position, it may prefer not to differentiate. This change highlights the importance of

[14] Beard (2013) documents several examples of how the spillover effect discourages negative advertising in practice: "Referring to comparative advertising that targeted prescription drugs Seldane and Alegra on behalf of Claritin, ad agency executive Lorraine Pastore tellingly told Advertising Age that they would not respond: "That would damage the category as a whole; its not a strategy we would be comfortable with." (Wilke, 1997)." Another example is by Microsoft's vice president of systems strategy, Jonathan Lazarus, who argues that negative advertising is "bad business. I don't think there's ever been a study that shows that negative advertising sells products. In our high-tech industry, people have a fear of the computer. They are worried about losing data and that it's complicated. So if I suddenly paint a competitor's products as complicated, I'm overall feeding those arguments that things will be tough to deal with" (Jaben, 1992).

studying a firm's product design and advertising strategies concurrently.

The existence of the equilibrium outcome described above requires a set of conditions to hold. We next discuss these conditions and the consumer beliefs about firm strategies that are consistent with them.

Inference from Advertising To simplify the discussion that follows, we will distinguish between firms based on their ability to advertise. We will refer to firm i as the "weak" opponent if $P_i = 0$ and $N_{-i} = 0$, i.e., if it cannot run any advertising. Otherwise, we will refer to the firm as the "strong" opponent. We will show that the optimal advertising (and consequently the consumers' beliefs about the optimal advertising) against a weak opponent is different from that against a strong opponent.

Under the PBE in Proposition 2, the consumers correctly believe that the firms prioritize (1) negative advertising (when locating apart) and positive advertising (when co-locating) against a strong opponent and (2) positive advertising (in either location) against a weak opponent. Let $\tilde{\mathscr{F}}(P_i, N_i, P_{-i}, N_{-i}) = \{P(P_i = \Pi), P(N_i = \beta), P(P_{-i} = \Pi), P(N_{-i} = \beta)\}$ stand for the posterior probability of each attribute being present in the products i and $-i$, given the observed advertising outcomes (a_i, a_{-i}), beliefs about which type of ads are prioritized by the firms. Then the posterior expectations about unobserved attributes of products mapping to each observed advertising outcome ($E[A_i|a_i, a_{-i}]$, $E[A_{-i}|a_i, a_{-i}]$) are as given in Table 1.[15]

The table demonstrates how the three effects of advertising choices by the firms influence consumer utility. First, the *direct effect* (in purple color) of advertising helps consumers update their beliefs about the advertised attribute. In particular, the prior beliefs put $P(P_i = \Pi) = \sigma_\Pi$, and positive advertising by firm i assures the consumers that product i has the positive attribute with certainty, resulting in the posterior belief $P(P_i = \Pi) = 1$ and an increase in the expected value $E[A_i]$ by $(1 - \sigma_\Pi)\Pi$. Similarly, the prior belief that product i has a negative attribute is $P(N_i = -\beta) = \sigma_\beta$. Negative advertising by the competitor firm $-i$ indicates that the product has the negative attribute with certainty, resulting in the posterior belief $P(N_i = -\beta) = 1$ and a decrease in $E[A_i]$ by $(1 - \sigma_\beta)\beta$. The direct effect does not vary with the position choice of the entrant.

Second, the *inference effect* (in black color) of advertising allows consumers to update beliefs about the unadvertised attributes of products in two different ways. One, if firm i runs no advertising, then consumers infer that product i must lack the positive attribute, resulting in the posterior

[15] Recall that prior expectation ($E[A_i]$) equals 0 due to the normalization $\sigma_\Pi \Pi = \sigma_\beta \beta$.

Locating Apart

a_i	a_{-i}	Prioritized Advertising i	$-i$	$\tilde{\mathscr{F}}(P_i, N_i, P_{-i}, N_{-i})$	$E[A_i]$	$E[A_{-i}]$
P_i	P_{-i}	N_{-i}	N_i	$\{1, 0, 1, 0\}$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$
P_i	N_i	N_{-i}	N_i	$\{1, 1, \sigma_\Pi, 0\}$	$(1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta$	$\sigma_\beta\beta$
N_{-i}	N_i	N_{-i}	N_i	$\{\sigma_\Pi, 1, \sigma_\Pi, 1\}$	$-(1-\sigma_\beta)\beta$	$-(1-\sigma_\beta)\beta$
P_i	$\varnothing$	P_i	N_i	$\{1, 0, 0, \sigma_\beta\}$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi$
N_{-i}	$\varnothing$	P_i	N_i	$\{0, 0, 0, 1\}$	$\sigma_\beta\beta - \sigma_\Pi\Pi$	$-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$
$\varnothing$	$\varnothing$	P_i	P_{-i}	$\{0, 0, 0, 0\}$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$

Co-location

a_i	a_{-i}	Prioritized Advertising i	$-i$	$\tilde{\mathscr{F}}(P_i, N_i, P_{-i}, N_{-i})$	$E[A_i]$	$E[A_{-i}]$
P_i	P_{-i}	P_i	P_{-i}	$\{1, \sigma_\beta, 1, \sigma_\beta\}$	$(1-\sigma_\Pi)\Pi$	$(1-\sigma_\Pi)\Pi$
P_i	N_i	P_i	P_{-i}	$\{1, 1, 0, \sigma_\beta(1-\rho) + \rho\}$	$(1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta$	$-\sigma_\Pi\Pi - \rho(1-\sigma_\beta)\beta$
N_{-i}	N_i	P_i	P_{-i}	$\{0, 1, 0, 1\}$	$-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$	$-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$
P_i	$\varnothing$	P_i	P_{-i}	$\{1, 0, 0, \sigma_\beta(1-\rho)\}$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi + \rho\sigma_\beta\beta$
N_{-i}	$\varnothing$	P_i	P_{-i}	$\{0, 0, 0, 1\}$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$
$\varnothing$	$\varnothing$	P_i	P_{-i}	$\{0, 0, 0, 0\}$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$

Table 1: Posterior Beliefs Given Advertising Outcomes. Note: Terms in purple indicate the "direct effect" of advertising, in black refer to the "inference effect," and in blue refer to the "spillover effect."

$P(P_i = \Pi) = 0$ and decreasing $E[A_i]$ by $\sigma_\Pi\Pi$. Moreover, consumers also infer that the competitor must lack the negative attribute, updating their posterior to $P(N_{-i} = -\beta) = 0$ and increasing $E[A_{-i}]$ by $\sigma_\beta\beta$. Two, if firm i runs an ad, then consumer inference depends on the position of the entrant and the type of opponent (weak vs. strong). In particular, if the entrant locates apart, consumers believe negative advertising is prioritized against strong opponents, consistent with the PBE described in Proposition 2. Hence, if firm $-i$ runs positive advertising, consumers infer that product i does not have the negative attribute, i.e., $P(N_i = -\beta) = 0$, increasing $E[A_i]$ by $\sigma_\beta\beta$. Otherwise, if the opponent is weak or the entrant co-locates, consumers believe positive advertising is prioritized. Hence, if firm i runs negative advertising, consumers infer that firm i does not have the positive attribute, i.e., $P(P_i = \Pi) = 0$, decreasing $E[A_i]$ by $\sigma_\Pi\Pi$.

Lastly, the *spillover effect* (in blue color) of advertising is only present if the entrant chooses to co-locate. On the one hand, when firm i runs negative and firm $-i$ runs positive advertising, consumers cannot learn about N_i through direct or inference effects.[16] However, consumers learn about the negative attribute of product $-i$ and update to $P(N_{-i} = -\beta) = 1$ due to firm i's negative advertising. Because the values of N_i and N_{-i} are correlated when firms co-locate, consumers take

[16] This is because in the Perfect Bayesian Equilibrium described in Proposition 2, firm $-i$ is expected to prioritize positive advertising regardless of the value of N_i. Therefore, consumers cannot make an inference.

the correlation between the product attributes into consideration, updating posterior beliefs to $P(N_i = -\beta) = (\sigma_\beta + \rho(1 - \sigma_\beta))$, which decreases $E[A_i]$ by $\rho(1 - \sigma_\beta)\beta$. In this case running negative advertising hurts the firm because consumers can deduce that the firm's own product is also likely to have the negative attribute. On the other hand, when firm i runs positive advertising and firm $-i$ does not advertise, consumers do not learn about N_{-i} through direct or inference effects. However, they update to $P(N_i = -\beta) = 0$ because of the absence of firm $-i$'s negative advertising. Again, because N_i and N_{-i} are correlated, consumers update their posterior beliefs to $P(N_i = -\beta) = \sigma_\beta - \rho\sigma_\beta$, which increases $E[A_{-i}]$ by $\rho\sigma_\beta\beta$. In this second case, avoiding negative advertising helps firm i because consumers can deduce that the product i is also likely to lack the negative attribute. In both cases, the presence of correlated attributes under co-location discourages negative advertising.

Below, we provide two examples to explain how the three effects of advertising work together to inform the consumer of product attributes.

Example 1. *Consider the second row in Table 1 under 'Locating Apart.' Here, after consumers observe firms advertising $a_i = P_i, a_{-i} = N_i$, they form beliefs about product attributes $\{1, 1, \sigma_\Pi, 0\}$. In this example, the direct effect of advertising reveals that $P_i = \Pi$ and $N_i = -\beta$. The former increases $E[A_i]$ by $(1 - \sigma_\Pi)\Pi$ while the latter decreases it by $(1 - \sigma_\beta)\beta$. Next, since firm i does not run negative advertising, which is prioritized under locating apart, consumers infer $N_{-i} = 0$. This inference effect increases $E[A_{-i}]$ by $\sigma_\beta\beta$.*

Next, consider the same advertising outcomes under 'Co-location' where consumers' beliefs about attributes are $\{1, 1, 0, \sigma_\beta(1-\rho)+\rho\}$. Recall that now positive advertising is prioritized. So while the direct effects of advertising remain the same as before, the inference effect changes. The absence of positive advertising by $-i$ leads consumers to infer that $P(P_{-i} = \Pi) = 0$, decreasing $E[A_{-i}]$ by $\sigma_\Pi\Pi$. This time, there is also a spillover effect to consider. Since consumers know that $N_i = -\beta$, they use this information to update beliefs about N_{-i} and conclude that firm $-i$ is more likely to have the negative attribute, which decreases $E[A_{-i}]$ by $\rho(1 - \sigma_\beta)\beta$. Notice that there is no spillover from positive advertising here. The direct and the inference effect of advertising fully reveal the values of positive attributes.

Example 2. *Consider the fourth row in Table 1 under 'Locating Apart.' Here, consumers observe firms advertising $a_i = P_i, a_{-i} = \varnothing$ and form beliefs $\{1, 0, 0, \sigma_\beta\}$. In this example, the direct effect of advertising reveals that $P_i = \Pi$, increasing $E[A_i]$ by $(1 - \sigma_\Pi)\Pi$. In addition, since firm $-i$ does not run any advertising, consumers infer $N_i = 0$ and $P_{-i} = 0$. While the former increases $E[A_i]$*

by $\sigma_\beta \beta$, the latter decreases $E[A_{-i}]$ by $\sigma_\Pi \Pi$.

Under 'Co-location,' consumers believe $\{1, 0, 0, \sigma_\beta(1-\rho)\}$. In this case, the direct and inference effects are identical to those under locating apart because positive advertising is, again, prioritized. However, the lack of firm $-i$'s negative advertising indicates $N_i = 0$, leading consumers to believe that firm $-i$ is less likely to have the negative attribute, increasing $E[A_{-i}]$ by $\rho\sigma_\beta\beta$. Similar to the previous example, there is no spillover from positive advertising.

3.2.1 Advertising Decision

After establishing the three effects of advertising, we next move on to analyzing advertising under different positioning decisions of the entrant. Through these three effects, advertising can help firms against their competitor and the outside option. In particular, advertising can help firms to steal consumers and protect own consumers from their opponent, and expand their market by reaching out to consumers with lower reservation values (γ_j). While positive advertising helps to improve its standing against the competitor and the outside option, negative advertising helps to improve the standing against the competitor but hurts it relative to the outside option. Firms will prefer to do negative advertising only if the number of consumers gained from the competitor exceeds those lost to the outside option.

In what comes next, given the belief set in Table 1, we will describe the conditions under which the equilibrium in Proposition 2 would hold based on a 2 × 2 environment description: locating apart vs. co-locating and competing against a strong vs. a weak opponent. We will see that the prevalence of a negative advertising equilibrium may vary depending on the environment.

Entrant Locates Apart If the entrant locates apart ($x_2 = R$), the consumers at R only buy from the entrant and the consumers at L only buy from the incumbent if both firms run the same type of advertising or they both do not run ads. To steal consumers from the opponent, the competitive advantage from advertising must surpass consumers' disutility from buying a product that does not match their preferences (measured by distance δ). For firm i to prioritize negative advertising, two conditions must be met. First, negative advertising must provide competitive advantage that exceeds the disutility from buying a less preferred product for the consumer, $E[A_i|a_i = N_{-i}, a_{-i}] - E[A_{-i}|a_i = N_{-i}, a_{-i}] > \delta$. Second, the competitive advantage from positive advertising must fall short of the same disutility, $E[A_i|a_i = P_i, a_{-i}] - E[A_{-i}|a_i = P_i, a_{-i}] \leq \delta$. In any other scenario (i.e., when these conditions cannot be met simultaneously) positive advertising is prioritized because running positive advertising increases $E[A_i]$ and expands the market, but negative advertising cannot steal more consumers from the opponent relative to positive advertising.[17]

Building on the above intuition, we next specify the conditions required to run negative advertising against a weak opponent in Lemma 1 and against a strong opponent in Lemma 2.

Lemma 1. (Locating Apart, Weak Opponent) *When firms locate apart, a firm running against a weak opponent prioritizes negative advertising if and only if $\beta > \delta \geq \Pi + \sigma_\beta \beta$. Otherwise, it prioritizes positive advertising.*

Recall that a weak opponent $-i$, by definition, cannot advertise (i.e., $P_{-i} = 0$ and $N_i = 0$). Then, when firm i runs either positive or negative advertising, the expected value of vertical attributes of firm i is higher, i.e., $E[A_i] > E[A_{-i}]$. When firm i runs negative advertising and $-i$ runs no advertising, $E[A_i] = \sigma_\beta \beta - \sigma_\Pi \Pi$, and $E[A_{-i}] = -(1-\sigma_\beta)\beta - \sigma_\Pi \Pi$. Hence, $E[A_i] - E[A_{-i}] = \beta$, so the first inequality in Lemma 1 ensures that negative advertising will allow stealing consumers from the opposite location. When firm i runs positive advertising instead and $-i$ runs no advertising, then $E[A_i] = (1-\sigma_\Pi)\Pi + \sigma_\beta \beta$, $E[A_{-i}] = -\sigma_\Pi \Pi$. Hence, $E[A_i] - E[A_{-i}] = \Pi + \sigma_\beta \beta$, so the second inequality ($\delta \geq \Pi + \sigma_\beta \beta$) ensures that positive advertising will not allow stealing consumers from the incumbent's location.[18] This is the condition stated in Lemma 1.

Lemma 2. (Locating Apart, Strong Opponent) *When firms locate apart, a firm running against a strong opponent prioritizes negative advertising if and only if (i) $\beta > (1-\sigma_\Pi)\Pi + \delta$ and (ii) $\bar{\gamma} + \sigma_\beta \beta \geq (1-\sigma_\Pi)\Pi + \delta$. Otherwise, it prioritizes positive advertising.*

Recall that a strong opponent $-i$ uses either positive or negative advertising. So for firm i to prioritize negative advertising against $-i$, two conditions must be satisfied. First, if the opponent runs positive advertising, firm i runs negative advertising only if it can steal the opponent's consumers. More specifically, condition (i) requires $E[A_{-i}|a_i = P_i, a_{-i} = N_i] - E[A_i|a_i = P_i, a_{-i} = N_i] > \delta$. From row 2 of Table 1 under locating apart, this inequality is equivalent to $\beta > (1-\sigma_\Pi)\Pi + \delta$. Condition (i) is necessary and sufficient to ensure that negative advertising is the best response to an opponent who runs negative advertising. While running positive advertising leads to 0 demand because it would allow the opponent to steal consumers, running negative advertising allows sharing the market equally with the opponent.

[17] To see this, recall that the model has the property that a firm either steals no consumers from the competitor or steals all consumers. Hence, once firm i achieves any competitive advantage that overcomes preference heterogeneity (i.e., $E[A_i] - E[A_{-i}] > \delta$), the magnitude of the competitive advantage becomes irrelevant.

[18] The set of parameters for which negative advertising is prioritized against weak opponents does not satisfy the remaining inequalities (i.e., (A1a)-(A1d) in the appendix) for the equilibrium in Proposition 2 to exist. Therefore, in the equilibrium described in Proposition 2, positive advertising is prioritized against a weak opponent when the firms locate apart.

Second, although condition (i) is necessary to ensure negative advertising is the best response to an opponent who runs positive advertising, it is not sufficient: firm i should acquire at least as many consumers from firm $-i$ as it would have acquired had it used positive advertising and expanded the overall market demand. Formally, $D_i(a_i = N_{-i}, a_{-i} = P_{-i}) \geq D_i(a_i = P_i, a_{-i} = P_{-i})$ must hold, where D_i is the total demand for firm i. Writing this inequality yields the condition $\bar{\gamma} + \sigma_\beta \beta \geq (1 - \sigma_\Pi)\Pi + \delta$ (please see the appendix for derivation). Jointly, conditions (i) and (ii) are sufficient to ensure that the number of consumers stolen from the opponent more than makes up for those lost to the outside option.

Entrant Co-locates In the absence of advertising, if the entrant co-locates ($x_2 = L$), all consumers are indifferent between the two products. Similar to the previous section, we will next describe the advertising decisions of firms when competing against a weak or a strong opponent.

Lemma 3. (Co-location, Weak Opponent) *When firms co-locate, a firm running against a weak opponent always prioritizes positive advertising over negative advertising.*

Since firms are otherwise symmetric, all that is required to steal an opponent's consumers is some advertising advantage (i.e., $E[A_i] - E[A_{-i}] > 0$). And because of the discrete nature of the consumer distribution, the number of stolen consumers is identical regardless of the size of this advertising advantage. Because both positive and negative advertising can generate this advantage against a weak opponent, firms always prioritize positive advertising against weak opponents under co-location, as positive advertising expands the overall market demand. This is summarized in Lemma 3.

Lemma 4. (Co-location, Strong Opponent) *When firms co-locate, a firm running against a strong opponent always prioritizes positive advertising if and only if* $\Pi > (1-\rho)(1-\sigma_\beta)\beta$. *Otherwise, negative advertising may be prioritized.*

For a firm to prioritize positive advertising against a strong opponent in the unique sub-game equilibrium under co-location, positive advertising should generate an advantage over negative advertising. If the opponent is strong, then, for positive advertising to be always prioritized, it must be effective enough to steal consumers if the opponent runs negative advertising: $E[A_i - A_{-i} | a_i = P_i, a_{-i} = N_i] > 0$, or equivalently $\Pi > (1 - \rho)(1 - \sigma_\beta)\beta$ must hold. This is the condition stated in Lemma 4. Notice that because a "spillover effect" is at work under co-location, the condition is more likely to be satisfied with a large correlation between the attributes (ρ). Put differently, when firms position similarly, and their attributes overlap more, the presence of a negative spillover makes negative advertising less desirable and curbs firms' desire to use this strategy.

3.2.2 Entrant's Positioning Choice

The discussion until now allowed us to describe the advertising choices of firms under both co-location and locating apart. Next, we discuss the entrant's positioning decision, which boils down to comparing the expected payoff from each position given firms' advertising and consumer beliefs. Since product attribute values realize after the entrant makes a positioning decision, the entrant calculates the expectation over all possible attribute realizations to derive the expected payoff from locating apart and co-locating with the incumbent.

In the PBE given in Proposition 2, the entrant has three considerations over the two location choices. First, locating apart allows the firms to serve a greater share of the consumers in the market. When the entrant co-locates with the incumbent ($x_2 = L$), consumers in R will incur a disutility of δ when buying from the firms located at L. Hence, a smaller fraction of consumers at R would make a purchase. Second, locating apart makes it more likely for the entrant to face a negative advertising attack because the spillover effect under co-location discourages negative advertising. Third, realizations where firms have similar attributes are more likely under co-location due to the correlation between product attributes. While this last consideration does not necessarily lead to a difference in payoffs, it can amplify or mitigate the magnitude of the previous two considerations. The main trade-off faced by the entrant is thus between the gains from serving a differentiated product and the losses from a negative advertising attack.

Lemma 5. (Location Choice) *Let the outcomes of the advertising sub-games be as given in Lemmas 1-4. The entrant co-locates if and only if*

$$\begin{aligned}(1-\sigma_\Pi)\sigma_\Pi\big((1-\rho)-(1-\sigma_\beta)(1-3\sigma_\beta)\big)\Pi \geq & \sigma_\beta(1-\sigma_\beta)\big(1-(1-\sigma_\Pi)(1-\rho)(1-\rho+\rho\sigma_\Pi)\big)\beta \\ & + \big(0.5-(1-\sigma_\beta)(\sigma_\beta+(1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\big)\delta\end{aligned}$$

The condition stated in Lemma 5 compares the expected payoffs for the entrant following co-location and locating apart. The condition suggests the entrant co-locates when (1) negative attributes are more likely to be present (high σ_β), (2) positive attribute is more valuable (high Π), and (3) consumer preferences are less heterogeneous (low δ). As σ_β grows, the negative advertising attacks become more likely; hence the entrant has a stronger incentive to avoid locating apart where negative advertising is prioritized. As Π grows, positive advertising leads to a bigger boost in demand; hence the entrant has a stronger incentive to co-locate where positive advertising is prioritized. The condition also suggests that as consumers' preference heterogeneity δ becomes larger, the advantage of serving a differentiated product becomes larger, and the entrant finds locating apart more desirable. Put differently, when consumers care about buying a product closer to their tastes, care more about the positive aspects of a product, and when products are more likely to have negative attributes, in equilibrium, product offerings are less differentiated, which may be less desirable from a consumer's perspective, as will be highlighted in Section 4.

3.3 Comparative Statics

We next discuss the characteristics of markets that are more likely to observe the equilibrium described in Proposition 2. Our analysis in this subsection takes the beliefs in Table 1 as given. We focus on $\{\delta, \rho, \sigma_\beta, \sigma_\Pi\}$: the dispersion in consumer tastes, the degree of correlation between the attributes of co-located products, and the prior beliefs about attribute values, respectively. The comparative statics on these parameters can map to various market conditions and shed light on when to expect negative and positive advertising to be more likely.

The parameter δ gives a simple measure of how dispersed the consumers are in their preferences for the horizontal attribute. In some sectors, consumers have strict preferences over what type of product they demand, while in others, consumers readily switch between different characteristics. Then, we might want to ask how the firm positioning depends on consumer taste heterogeneity, given the threat of negative advertising.

Corollary 1. (Degree of Heterogeneity in Consumer Tastes) *There exists a threshold $\bar{\bar{\delta}}$ for the consumer taste heterogeneity such that the entrant co-locates only if $\delta \leq \bar{\bar{\delta}}$. The entrant always locates apart for $\delta > \bar{\bar{\delta}}$.*

When consumer taste heterogeneity is low, negative advertising becomes more attractive for firms located apart, as it allows stealing customers located at the other end of the line. To avoid a negative advertising war, firms have stronger incentives to co-locate. Johnson and Myatt (2006) also concluded that a smaller taste dispersion would lead to a more generic product design because firms want to be able to market to a larger audience. Our analysis adds one more mechanism in line with this result: the incentive to avoid a negative advertising war.

The parameter ρ measures the likelihood that the vertical attributes will be similar for co-located products. We next analyze the effect of attribute correlations on product positioning.

Corollary 2. (Degree of Correlation) *There exist thresholds $\bar{\rho}$ and $\underline{\rho}$ for the correlation between product attributes such that co-located firms prioritize positive advertising only if $\rho \geq \underline{\rho}$. The entrant co-locates only if $\underline{\rho} \leq \rho \leq \bar{\rho}$.*

The first part of Proposition 2 indicates that when the correlation among attributes is high, it acts as a deterrent to negative advertising when firms are co-located. The second part indicates that an entrant is less likely to co-locate when ρ is too large or too small. If the correlation is too small ($\rho < \underline{\rho}$), then co-location is followed by firms prioritizing negative advertising; hence there are no incentives to co-locate. If the correlation is too high ($\rho > \bar{\rho}$), then it becomes less likely that only one firm will have the positive attribute and have the whole market to itself. This reduces the expected payoff from co-locating and hence reduces the incentives to do so.

Finally, parameters σ_Π and σ_β indicate the prior probability that the products have positive and negative attributes, respectively. On the one hand, as σ_Π (σ_β) grows, positive (negative) advertising becomes less effective since it leads to only a marginal update in consumer beliefs—or, the "direct effect" of advertising becomes smaller. On the other hand, when σ_Π (σ_β) grows, the absence of a positive (negative) attribute results in a large update in consumer beliefs—or, the "inference effect" of advertising becomes larger. Corollary 3 summarizes how changes in σ_Π and σ_β impact advertising decisions.

Corollary 3. (Prior Beliefs about Product Attributes) *There exist thresholds $\underline{\sigma_\Pi}$ and $\underline{\sigma_\beta}$, such that when the presence of both attributes is more likely ($\sigma_\Pi > \underline{\sigma_\Pi}$ and $\sigma_\beta > \underline{\sigma_\beta}$), competing firms are more likely to engage in positive (negative) advertising under co-location (locating apart).*

The intuition of Corollary 3 follows from the fact that as σ_Π and σ_β grow together, the "direct effect" of advertising shrinks while the "inference effect" grows. With a large "inference effect," consumer's beliefs can be self-fulfilling: firms may find a strategy to be optimal because consumers believe that only a firm whose product has desirable attributes would take the associated action. Therefore, the higher σ_Π and σ_β are, the more likely are firms to run advertising in accordance with consumer beliefs given in Table 1.

3.4 Discussion of Multiplicity of Equilibria

Our model admits a large number of PBE that can be supported with carefully constructed beliefs, similar to other signaling games. To understand why, fix the location choices and suppose, first, consumers believe that positive advertising is prioritized when both negative and positive advertising are feasible ($P_i = \Pi, N_{-i} = -\beta$). If the firm does not run positive advertising, consumers infer that the positive attribute is missing. Then, a firm may prioritize positive advertising to avoid being perceived as a bad type ($P_i = 0$). Now suppose that consumers believe negative advertising is prioritized when both are feasible. If the firm does not run negative advertising, consumers infer that the negative attribute is missing from the competitor. Then, a firm may prioritize negative advertising to prevent the competitor from being perceived as a good type ($N_{-i} = 0$). Hence, consumer beliefs can be self-fulfilling and different advertising equilibria can be observed in otherwise identical markets.

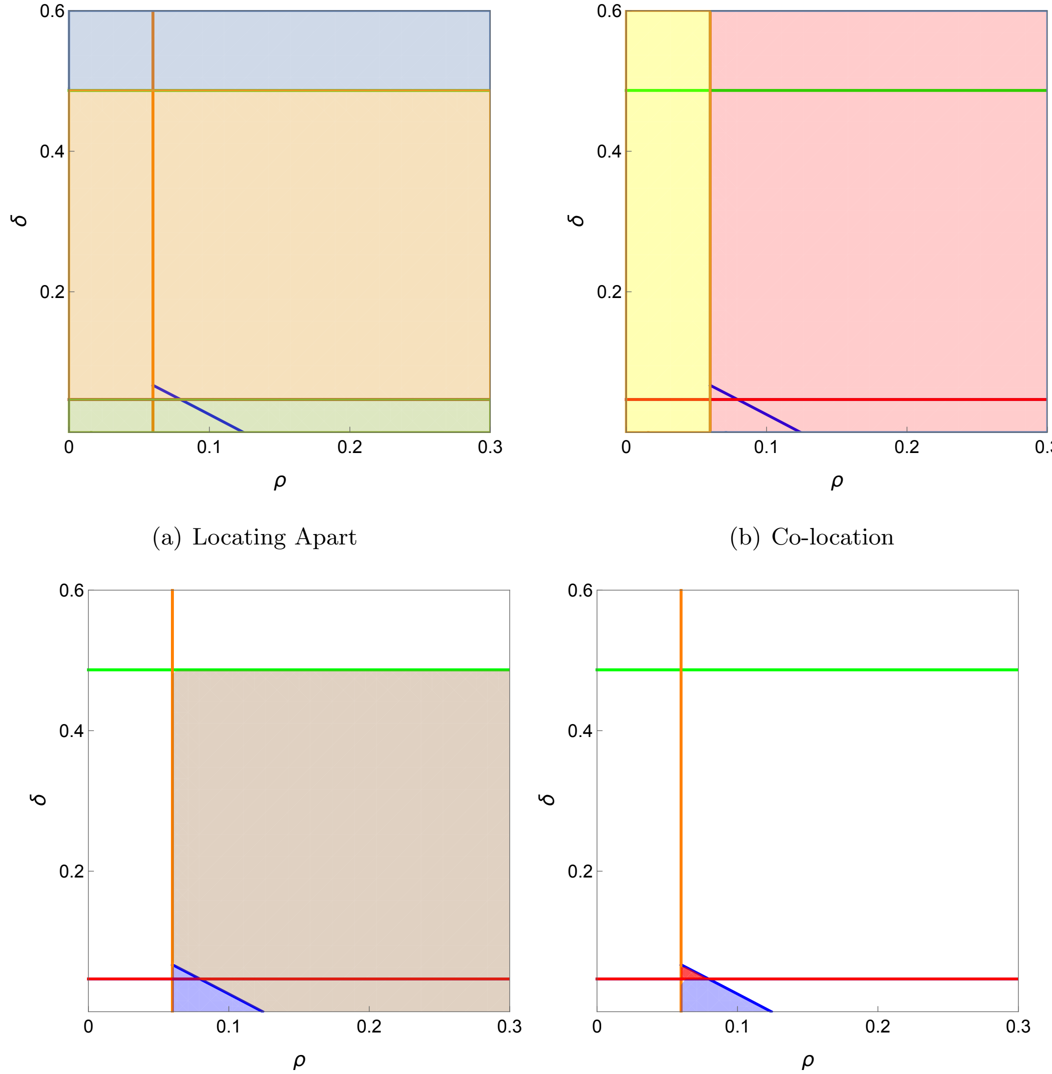
Nevertheless, we can eliminate some equilibria that are less intuitive than others. For a given set of beliefs, there are certain parameter sets where the advertising *sub-games* have multiple equilibria: (i) one where both firms prioritize positive advertising, (ii) one where both firms prioritize negative advertising, and (iii) a mixed-strategy equilibrium where firms prioritize each with a non-zero probability.[19] Pareto (or payoff) dominance, as an equilibrium selection criterion, suggests favoring equilibria whose payoffs Pareto-dominate the payoffs from other equilibria. One intuition behind its appeal is that non-binding pre-play communication would rule out Pareto-dominated equilibria. The payoffs in equilibria of type (i) Pareto-dominate equilibria of types (ii) and (iii) for both firms. Hence, a refinement *à la* Harsanyi and Selten (1988) allows eliminating equilibria of type (ii) and (iii).

After implementing these selection criteria, there remain five equilibria characterized by which advertising is prioritized after each location choice and entrant's product choice. We will use the notation $CO_{\hat{a}(Co-locate)\hat{a}(LocateApart)}$ to denote an equilibrium where the entrant chooses to co-locate and $LA_{\hat{a}(Co-locate)\hat{a}(LocateApart)}$ to denote an equilibrium where the entrant chooses to locate apart, given $\hat{a}(.) \in \{P, N\}$, which denotes the prioritized advertising (against strong opponents) following the location choice. For example, the equilibrium introduced in Proposition 2 is denoted as CO_{PN}, i.e., the entrant co-locates, positive advertising is prioritized after co-location and negative advertising is prioritized after locating apart. The other four equilibria that survive our elimination are LA_{PN}, LA_{PP}, LA_{NN}, and LA_{NP}. See that the only advertising outcome that can incentivize co-location is the equilibrium we introduced in Proposition 2.

It is natural to ask which parameter sets map to which of each remaining equilibria. We start by visualizing the parameter sets where firms prioritize positive vs. negative advertising after each location choice. For these plots, we fix the values of other parameters and vary taste heterogeneity

[19]Multiple sub-game equilibria arise when negative advertising is sufficient to steal consumers from the opponent, but the number of stolen consumers does not make up for the lost demand due to shrinking market size. In such a situation, the best response to a firm that prioritizes positive (negative) advertising is to prioritize positive (negative) advertising.

(δ) and correlation between attribute values (ρ).[20] In Figure 3, four constraints define the regions of equilibria, as given in each subfigure.



(a) Locating Apart

(b) Co-location

(c) Location Choice

(d) Equilibrium in Proposition 2

Figure 3: Equilibria Regions ($\beta = 1, \sigma_\Pi = 0.3, \sigma_\beta = 0.22, \bar{\gamma} = 1$)

Subfigure (a) visualizes parameter sets that can sustain different advertising equilibria following locating apart. In particular, if taste heterogeneity (δ) is sufficiently large (light blue region), negative advertising never pays off, even if consumers were to believe it is prioritized. Hence, the

[20] To simplify things, we choose $\bar{\gamma}$ sufficiently large so that if firms can steal consumers from the opponent with negative advertising, the number of consumers stolen is always sufficiently large (relative to the number of those lost to the outside option) to justify negative advertising. The rest of the parameter values are chosen to visualize as many relevant regions as possible.

only equilibria that exist are those that prescribe positive advertising following locating-apart. In contrast, if taste heterogeneity (δ) is sufficiently low (green region), negative advertising is always sufficient to steal consumers, even if consumers were to believe positive advertising is prioritized. Hence, the only equilibria that exist are those that prescribe negative advertising following locating apart. The region in between (orange region) can sustain equilibria that prescribe either type of advertising. Thus, LA_{PN}, LA_{NN}, and CO_{PN} does not exist in the light blue region, while LA_{PP} and LA_{NP} do not exist in the green region. The intution for these findings closely follows Corollary 1.

Subfigure (b) in Figure 3 visualizes parameter sets that can sustain different advertising equilibria, following co-location. In particular, if the correlation between attribute values (ρ) is sufficiently small (yellow region), negative advertising cannot be avoided, even if consumers were to believe that positive advertising is prioritized. Hence, the only equilibria that exist are those that prescribe negative advertising following locating-apart. In the remaining pink region, ρ would be sufficiently large to discourage negative advertising if consumers believe positive advertising is prioritized, but also sustain negative advertising if consumers instead believe negative advertising is prioritized. Thus, LA_{PN}, LA_{PP}, and CO_{PN} does not exist in the yellow region, while all five equilibria exist in the pink region. The reasoning closely follows Corollary 2.[21]

Subfigure (c) visualizes the location choice of the entrant, fixing the advertising equilibrium to prescribe positive advertising under co-location and negative advertising under locating apart. In the brown region, with higher δ and ρ values, firms locate apart, and in the purple region where both δ and ρ values are rather small, firms choose to co-locate. Hence, LA_{PN} only exists in the brown region while CO_{PN} only exists in the purple region.

The last subfigure, (d), is representing the region where the equilibrium described in Proposition 2 exists. This region is divided into two—the upper part of the triangle (in red) can sustain LA_{PP}, LA_{NP}, and LA_{NN} as well as CO_{PN}, while the lower part (in blue) only sustains LA_{NN} together with CO_{PN}.

Figure 3 does not provide a region where the equilibrium described in Proposition 2 (CO_{PN}) is unique. In Proposition 3 we formally prove that, indeed, there does not exist a parameter set where this equilibrium is unique.

Proposition 3. (Positioning and Advertising — Uniqueness) *There is no set of parameters for which the PBE in the full game, where the entrant co-locates and both firms prioritize positive*

[21] For the fixed parameter values that we have chosen, there is no region where firms always do positive advertising, even if consumers were to believe that negative advertising is prioritized. However, this region can arise for $\beta < \Pi$.

advertising, is unique.

Proposition 3 indicates that the regions that support the equilibrium introduced in Proposition 2 also support other equilibria. This is because spillovers from negative advertising are replaced by spillovers from positive advertising if negative advertising is prioritized under co-location. Hence, parameters that can sustain negative advertising under locating apart can also sustain it under co-location.

4 Welfare Analysis

We next turn our attention to the welfare gains for consumers from negative and positive advertising. On the one hand, negative advertising allows consumers to learn about the attributes of own and competitor's products, resulting in an "information gain." On the other hand, the threat of negative advertising reduces the assortment of products in a market, therefore resulting in a "loss in product match." We compare consumer welfare from the benchmark equilibrium when negative advertising is not permitted (in Proposition 1) to the equilibrium when negative advertising is feasible (in Proposition 2). We present the results in Lemmas 6 and 7.

Lemma 6. (Welfare Gain due to Information) *Permitting negative advertising leads to no additional information revealed to consumers when both products have the positive attribute. If one or both products lack the positive attribute, however, consumers have welfare gains due to additional information from negative advertising. The associated welfare gain increases with the uncertainty around the negative attribute ($\sigma_\beta(1-\sigma_\beta)$).*

Lemma 6 follows from the fact that in a market with negative advertising, both the presence and the absence of negative advertising can be informative to a consumer. When firm i engages in negative advertising, it informs the consumer of the competing product's (direct effect) or its own product's weak characteristics (spillover effect). Similarly, if firm i does *not* engage in negative advertising, it informs the consumer about the absence of the competing product's weak characteristics (inference effect) or the absence of the own product's weak characteristics (spillover effect). In both cases, the presence of negative advertising may increase consumer welfare by convincing consumers to buy a product with high valuation, by helping consumers avoid buying a product with low valuation, or by helping them find the product that better fits their taste. The only situation where the absence of negative advertising does not inform the consumers is when both products have the positive attribute. This is because firms prioritize positive advertising, and hence consumers do not update their prior on the negative attribute.

Lemma 7. (Welfare Loss due to Reduced Product Differentiation) *If only one product has the positive attribute, permitting negative advertising leads to no change in welfare through product differentiation. Otherwise, permitting negative advertising leads to welfare loss through reduced product differentiation. The associated welfare loss increases in consumers' taste heterogeneity (δ).*

Lemma 7 presents an outcome indicating a possible welfare loss due to the reduced differentiation or variety of products in a market when negative advertising is allowed. In Proposition 2, we highlighted that due to the threat of a negative advertising war down the road, an entrant coming to a market chooses to co-locate with an incumbent, reducing product variety in the marketplace. This reduced variety implies that, for some consumers, there will be a loss of welfare due to either purchasing a product that is different from their ideal product or due to not buying any products at all. Unlike the welfare gains due to information, the welfare loss discussed here is not a direct outcome of learning about advertised attributes, but an indirect outcome of the product design choices made by firms which anticipate negative advertising possibility.

In addition to the above two effects, allowing negative advertising results in an additional effect, as it leads to co-location in Proposition 2. The joint distribution of attributes is different under co-locating compared to locating apart due to the presence of a correlation (ρ) between attributes. Thus, there is a "correction" term in the welfare change which can take a positive or a negative value, is proportional to ρ, and disappears as $\rho \to 0$.

We evaluate the net change in welfare due to the above-discussed three channels in the proposition that follows.

Proposition 4. ***(Consumer Welfare)*** *When the prior uncertainty around the negative attribute ($\sigma_\beta(1-\sigma_\beta)$) is sufficiently small, permitting negative advertising leads to a loss in consumer welfare. This loss increases in consumers' taste heterogeneity (δ).*

Proposition 4 states that, in markets where the prior uncertainty around the negative attribute is small—or there is sufficient prior information about negative attributes—permitting negative advertising leads to a welfare loss. Furthermore, as consumers care more about purchasing a product that is close to their ideal one, permitting negative advertising leads to a bigger loss of welfare. While consumers make a more-informed product choice, the threat of negative advertising reduces the variety of the products that they can buy. In particular, the products that consumers purchase may match their preferences poorly when negative advertising is allowed. This finding emphasizes the adverse welfare consequences of unregulated advertising for consumers. While some negative effects from advertising competition have been documented (Fruchter, 1999; Stegeman, 1991), the effects of advertising on product variety and product design choices have not been investigated, despite the critical consequences for consumers.

5 Extensions

In this section, we extend our model to two new settings. First, in the main model, we assume that firms are price-takers and here, relaxing this assumption, we consider the implications of price competition. Second, we consider the application of negative advertising to an alternate environment: political marketing.

5.1 Pricing with Consumer Search

In the main model, to improve tractability and exposition, we assumed that competing firms in the market are price-takers. This assumption may not be without loss of generality when introducing dissimilar products, as it may alleviate price competition between firms. Then, pricing might act as a counter-balancing force to the co-location-inducing effect of negative advertising. Our aim in this section is to show that even though co-location leads to higher price competition, if this competition is not extreme, then the results from our main model hold qualitatively.

Adding pricing is not straightforward, since price competition *à la* Bertrand would lead to a zero price under co-location. To soften competition, we introduce a search cost for consumers, using a similar approach to that in Diamond (1971) and Kuksov (2004). In this model, consumers search for price quotes about competing products and can only purchase a product after receiving a price quote. Each consumer can access the first price quote for free, but has to pay an additional cost κ to receive a second price quote. Firms' pricing decisions and consumers' search decisions are made simultaneously.

With this model, we make three modeling modifications to incorporate prices. First, the (indirect) utility function of consumers is now

$$U_{ij}^{p} = \gamma_j + A_i - |x_i - \chi_j| - p_i, \tag{3}$$

where p_i denotes the price of good i. Second, consumers face search frictions as in Diamond

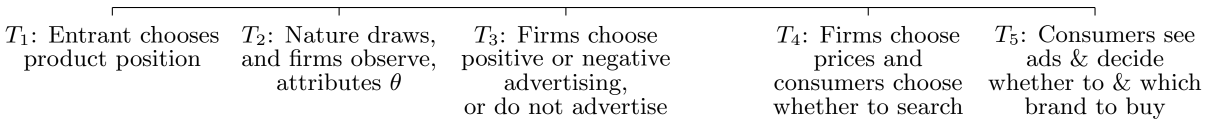


Figure 4: Timeline of the Incumbent-Entrant Game with Pricing

(1971). When two products are otherwise symmetric for consumer j, i.e, $E[A_i] - |x_i - \chi_j| = E[A_{-i}] - |x_{-i} - \chi_j|$, then consumers are equally likely to receive the quote of either product for free.[22] When $E[A_i] - |x_i - \chi_j| > E[A_{-i}] - |x_{-i} - \chi_j|$, then consumers receive the price quote for product i for free. This structure resembles the setting in Kuksov (2004) and fits well to a market where consumers have easier access to advertising and information about product characteristics relative to price information.[23] Third, with the modifications given above, we assume that firms engage in price competition to maximize revenues after deciding on their product positioning and advertising strategies. Consumers decide whether to search for the second price quote, whether to buy a product, and if they do, which product to buy to maximize their indirect utility. The updated timing of the game is reflected in Figure 4.

In this model, prices do not provide an additional signal regarding the value of the attributes. This is because a firm's optimal pricing strategy depends on the actual values of the attributes only through consumers' posterior beliefs after they see the ads. To solve the model under price competition, we update the equilibrium definition given on p. 13 as follows.

Definition A PBE with pricing is the positioning decision of the entrant $x^2 \in \{L, R\}$, advertising $a^2(x_2, \theta) \in \{P_2, N_1, \varnothing\}$, $a^1(x_2, \theta) \in \{P_1, N_2, \varnothing\}$ and pricing decisions $p^2(x_2, a_1, a_2, \theta) \in R^+$, $p^1(x_2, a_1, a_2, \theta) \in R^+$ of firms, beliefs of consumers over firm types $\mathscr{F} : \theta_1 \times \theta_2 \to [0, 1]$, search $\{s^j(x_2, a_1, a_2, \theta)\}_j \in \{search, not\} \in R^+$ and purchase decisions of consumers $\{g^j(x_2, a_1, a_2, p_1, p_2, s^j)\}_j \in \{i, k, o\}$ such that

1. Consumers' purchase decisions are sequentially rational, i.e., $\{g^j(.)\}_j$ maximizes $E[U^p_{ij}|\mathscr{F}]$.

2. $p^2(.)$, $p^1(.)$, and $\{s^j(.)\}_j$ constitute a Nash equilibrium of the pricing-search sub-game given $\{g^j(.)\}_j$.

3. $a^2(.)$ and $a^1(.)$ constitute a Nash equilibrium of the advertising sub-game given $p^2(.)$, $p^1(.)$, $\{s^j(.)\}_j$, and $\{g^j(.)\}_j$.

[22] Kuksov (2004) follows a similar structure and provides a detailed description of the implications of assuming search costs for prices.

[23] See Christou and Vettas (2008) for a model where advertising informs consumers about prices instead.

4. The positioning decision $x^2 \in \{L, R\}$ maximizes firm 2's profits given $a^2(.)$, $a^1(.)$, $p^2(.)$, $p^1(.)$, $\{s^j(.)\}_j$, and $\{g^j(.)\}_j$.

5. The consumers' beliefs $\mathscr{F}$ are updated based on $a^2(.)$ and $a^1(.)$ according to the Bayes' Rule.

Lemma 8. ***(Pricing Strategies)*** *Consumers do not search for a second quote in the advertising sub-games. Under locating apart, when the advertising outcomes are symmetric ($a_1 = a_2$), firms charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2}$. Otherwise, there exists a $\bar{\delta}$ such that firms charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2} - \frac{\delta}{4}$ for $\delta \leq \bar{\delta}$, and charge $p^i = \frac{E[A_i]+\overline{\gamma}}{2}$ for $\delta > \bar{\delta}$.*

Lemma 8 suggests that in a game with pricing, independent of firms' location choices, consumers do not search for a second price quote. As a result, the prices in the market resemble the prices under a monopoly, as if there was no competition.[24]

The associated price depends on the location and advertising outcomes. When firms locate apart and $a_1 = a_2$, the incumbent sells to consumers at position L and the entrant sells to consumers at position R. In this case, the firms price as "local" monopolies. In any other scenario, the firm(s) sell to both locations, hence price as a "global" monopoly. Since the global monopoly has to overcome the taste heterogeneity δ to reach to consumers at the more distant location, the associated revenue-maximizing price is lower. Hence, the price is weakly lower under co-location relative to locating apart, and the difference is proportional to the consumer taste heterogeneity (δ). Furthermore, better advertising outcomes (high $E[A_i]$) increase the revenue-maximizing prices.

Proposition 5. (Positioning and Advertising with Pricing) *Pricing competition reduces the parameter region where firms co-locate in equilibrium. However, when δ is sufficiently small, the PBE described in Proposition 2 still exists after including price competition.*

Proposition 5 delivers the key point that there still exists a PBE as described in Proposition 2: the entrant co-locates and both firms prioritize positive advertising and in the off-equilibrium path where the entrant locates apart, firms engage in a negative advertising war, even when positive advertising is an available option. In other words, our results are robust to the inclusion of a price competition *a là* Diamond, and the entrant may still choose to co-locate with the incumbent to avoid a negative advertising war. However, as anticipated, co-locating puts a downward pressure on prices, and relative to the benchmark scenario where firms were price-takers, co-locating is less likely.

[24] This is the well-known paradox of Diamond (1971): regardless of how small the search cost is, as long as it is positive, the market behaves like a monopoly.

5.2 Political Positioning and Advertising

Our results so far suggest that firms have incentives to avoid negative advertising competition, and this might even induce them to avoid differentiating. The reader may agree that, indeed, many firms do not engage in negative advertising, even though it is legal. In political competition, however, negative advertising is utilized frequently (Ansolabehere et al., 1994). As these campaigns are among the most sophisticated marketing campaigns in the US (Petrova et al., 2021), readers might ask why then we observe increased differentiation accompanied by extensive negative advertising wars in political competition. In this section, we extend our model to study negative advertising in politics. This section will demonstrate that since political candidates do not care about a drop in voter turnout if all candidates lose voters proportionally, they tend to run negative advertising more often than firms do.[25] [26]

We choose the political competition setting as close to our setting for commercial firms as possible. Most of the model components are identical, except for the interpretation and the labels—consumers are replaced by "voters" and firms are replaced by "political candidates." There is an incumbent politician and an entrant (challenger) running for the office. While the position (e.g., policy stance) of the incumbent is already set, the entrant can choose her position. The only difference we introduce here compared to the benchmark model in Section 2 is regarding the objective of the candidate: to maximize the *winning probability* instead of the number of votes.[28] We demonstrate below that this simple difference in the objective function goes a long way to account for the tone differences in political advertising. Since the only difference is in the objective function, the action sets and equilibrium definitions are identical to those in the product competition described in the benchmark.

In the next section, we discuss how the incentives for negative advertising change under political competition.

[25] Results are provided for a two-candidate political race. Empirical studies show that in the U.S., indeed, most elections are bipartisan (Garcia-Jimeno and Yildirim, 2017).

[26] While we assume that politicians are truthful, it should be noted that a key difference between commercial and political advertising is the applicability of truth-in-advertising regulations. While the FTC has the power to pull any misleading or incorrect ads, it does not have jurisdiction over "speech," which is protected by the first amendment. Political advertising is considered political speech, and as a result, FTC's truth-in-advertising regulations do not apply to political advertising.[27]

[28] We assume that when the candidates have an equal number of votes, the winning probability equals 0.5 for both candidates.

5.2.1 Advertising in Political Competition

Changing the objective from maximizing the number of votes to maximizing the winning probability has a direct effect on candidates' advertising incentives. In particular, in product competition, advertising that reduces the demand for both firms would not be used, but it can be used in political competition as long as it reduces the opponent's vote count more than it reduces the own vote count. Thus, the main disadvantage of negative advertising under product competition—that it shrinks the overall market—is not a disadvantage in political competition. Proposition 6 shows how the change in the objective function can explain the prevalence of negative advertising in political competition.

Proposition 6. (Political Advertising) *The set of parameters for which negative advertising is prioritized in product competition is a strict subset of the set of parameters for which negative advertising is prioritized in political competition. Put differently, negative advertising is weakly more likely in political competition than in product competition.*

Proposition 6 indicates that, the incentive to run negative advertising is higher compared to political competition. To see this, first consider why the incentives to run negative advertising against a weak opponent are stronger. Facing a weak opponent, both positive and negative advertising result in a higher advertising impact $E[A_i]$ than the opponent. In product competition, this results in positive advertising being prioritized, regardless of consumer beliefs: if consumers won't buy from the competitor, the firm should only focus on increasing its own $E[A_i]$. In political competition, however, negative advertising may still be prioritized as long as its negative effect on the opponent is sufficiently stronger than the effect of positive advertising on own. In product competition, under locating apart, a necessary condition for negative advertising to be prioritized is that only negative advertising allows stealing consumers from the competitor, i.e., $E[A_i - A_{-i}|a_i = N_{-i}] > \delta \geq E[A_i - A_{-i}|a_i = P_i]$. In political competition, negative advertising may be prioritized even when it does not result in stealing votes. The only requirement to prioritize it is that, it leads to a higher winning probability, i.e., $E[A_i - A_{-i}|a_i = N_{-i}] \geq 0$, which is a weaker condition than the one in product competition.

Second, consider the incentives to run negative advertising against strong opponents. In product competition, under both co-location and locating apart, two conditions need to be satisfied for negative advertising to be prioritized: (i) A firm utilizing negative advertising should be able to steal consumers from a positive advertiser, and (ii) the number of consumers stolen should make up for those who are lost to the outside option. In political competition, condition (ii) is not necessary because a decline in the own vote count is not damaging as long as the opponent's vote count declines more. Furthermore, a weaker version of condition (i) is sufficient: $E[A_i - A_{-i}|a_i = N_{-i}] \geq 0$ may be desirable even when it doesn't allow stealing voters. We discuss the implications of these altered incentives for the positioning decision of a political entrant next.

5.2.2 Positioning Strategies in Political Competition

Although the model has sharp predictions about advertising in political competition, it has less to say about the positioning choices of new candidates. However, this result changes sharply if we assume that a mass η of voters at point L ("core supporters") slightly prefer the incumbent more.[29] The presence of supporters could be explained by the well-documented incumbency advantage literature (Petrova et al., 2021). In political competition, this tweak would immediately pin down the equilibrium. A newcomer would locate apart as co-location leads to a defeat even after symmetric advertising decisions. In product competition, however, an entrant might still choose to co-locate to escape a negative advertising war when η is sufficiently small, as demonstrated in Proposition 7.

Proposition 7. (Positioning in Political vs. Product Competition) *In political competition, (i) when $\eta = 0$, the entrant is always indifferent between co-locating and locating apart, regardless of the advertising sub-game outcome, and (ii) when an arbitrarily small mass η of voters at L prefer the incumbent, the entrant always chooses to locate apart. In product competition, the entrant firm may co-locate when η is sufficiently small.*

Part (i) of the proposition suggests that, in a political competition where voters are symmetrically distributed in their preferences, either location choice by the incumbent results in an (expected) winning probability of $\frac{1}{2}$. Hence, the rich feedback from advertising incentives to positioning that exist in product competition are missing in political competition. In this case, advertising is inconsequential for the entering politician's positioning choice. However, with a small (and intuitive) tweak in voter preferences, we get much sharper predictions about positioning in political competition.

The intuition behind part (ii) of the proposition is that, while a few consumers have a small impact on the outcome of product competition, a few voters can determine the winner in political competition when candidate attributes are similar. In product markets, the fear of negative advertising wars might push an entrant into a product choice where the consumer base is already favoring the incumbent. In politics, negative advertising wars are nothing to be afraid of for candidates, yet

[29] $U_{ij} = \gamma_j - |x_i - \chi_j| + A_i - \mathbf{1}_{\{\chi_j = L\}}\epsilon$ where $\epsilon > 0$ and i is the entrant.

a few votes can change the outcome when the candidates are similar. This result corroborates the findings from studies suggesting significant polarization in political races (Gentzkow et al., 2019).

This section emphasizes how changing objectives in product and political advertising leads to two very different outcomes. Because candidates are not necessarily hurt by a shrinking voter base, unlike firms do with a shrinking consumer base, they are more likely to take the risk and engage in negative advertising.

6 Conclusions and Discussion

Negative advertising is a form of advertising that informs and persuades consumers about the weaknesses of a competitor's product, and through that it highlights the relative advantage of one's own product. While negative advertising is commonly utilized, its implications are little understood. This study focuses on the competition between two substitute products and their product design strategies in anticipation of a negative advertising war. The firms face a trade-off between choosing sufficiently differentiated designs that allow matching consumer heterogeneity and similar designs that pre-empt negative advertising.

In this setting, we find that the threat of negative advertising can motivate firms to co-locate, reducing product differentiation in the marketplace. While co-locating reduces the likelihood of a negative advertising attack down the road, it also leads to under-utilization of the full consumer demand. Moreover, permitting negative advertising may result in an overall welfare loss for consumers when the loss from reduced product differentiation exceeds the gain from additional information that consumers receive.

In an extension, we study the case of political competition, where the objective of competitors is to win by plurality (and not profit maximization). We show that this change in the objective function can explain the relatively widespread use of negative advertising in electoral races, as competitors care less about an overall reduction in voter turnout.

Our findings have important implications for managers and policymakers alike. For managers, our study highlights the close relationship between product design and advertising wars. As explained in detail in the introduction, negative advertising wars reduce demand for all involved parties. Therefore, a firm entering a market may want to think if its product's design features will risk negative advertising. While regulators shunned negative advertising in various parts of Europe until the 1990s,[30] in the U.S., the FTC's Regulatory Overboard of Advertising encourages firms to name their competitors and draw comparisons about pricing and product attributes.[31] The FTC argues that comparative advertising "assist [consumers] in making rational purchase decisions" through direct comparisons of brands and "encourages product improvement and innovation, and can lead to lower prices in the marketplace." While comparative advertising includes other forms than negative advertising, the regulations regarding the mention of a competitor define if and how negative advertising can be utilized by firms. Therefore, FTC's encouragement of comparative advertising may also be read as an encouragement for negative advertising, resulting in firms having less differentiated product designs to avoid subsequent negative advertising wars in some sectors. Put differently, in contrast to the FTC's claims, product improvement and innovation may be discouraged due to the threat of negative advertising.

Moreover, for negative advertisements to inform consumer decision-making and incentivize firm innovation, firms should be actually using it in practice. Despite the FTC's encouragement, only about one-third of commercial brands ever engage in comparative ads (Grewal et al., 1997). Our model shows that regulations restricting negative advertising can be welfare improving.

While, to our knowledge, this paper is the first to study negative advertising in the context of product design, we have kept our model intentionally simple to deliver clear and important insights. Our framework can be enriched in various ways. First, each firm's advertising may be received by some (not all) customers, and this subset of customers may be chosen randomly or may be targeted by the firm. This may reduce the harm from negative advertising and reduced differentiation. Second, our setting has a timing assumption wherein firms have limited opportunities to learn about consumers' appreciation of attributes and consumers learn about the products through advertising. It is possible that firms learn about consumer valuation and revisit their design and advertising strategy multiple times, or external events may reveal additional information about the attributes, resulting in consumers updating their valuations. Our model does not explicitly consider these dynamics; however, future research may want to expand to these settings. Third, we took a simplistic approach to account for the role of prices. Alternate models of advertising can be written to explicitly account for price competition or use prices as a signal in consumer inference about product attributes. Our model can be extended to account for empirical results regarding how other (if any) competitors benefit from negative advertising wars between two competitors (see

[30] In EU, until 2000, mentioning competitor's name in advertising was regarded as an improper business practice, and trading on another firm's reputation and goodwill was considered unfair (Romano, 2004).

[31] https://www.ftc.gov/public-statements/1979/08/statement-policy-regarding-comparative-advertising

Anderson et al. (2016), Gandhi et al. (2016) and Galasso et al. (2020)). Finally, we summarize three ideas with the hope that other scholars can build on them. First, our study highlights the idea that negative advertising shrinks overall industry demand. While a very important subject, there has been little empirical investigation on this issue by marketers, and future research can contribute to this area. Second, we find that a change in a regulation that permits negative advertising might lead to reduced product innovation. Since such a regulation change was experienced in various EU countries, empirical researchers can put this finding to test by collecting data from numerous products over multiple years. Third, and in a similar vein, we find that there can be a consumer welfare loss from permitting negative advertising; and empirical studies can also focus on this question.

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ONLINE APPENDIX

A.1 Discussion: Simultaneous Entry of Competing Firms

In the main model, we demonstrated that the desire to avoid a negative advertising war can cause an entrant to choose a product positioning that is similar to the existing offering in the market. Next, we consider the case where two competing firms enter a new market and choose their positions simultaneously.

Definition A Perfect Bayesian Equilibrium (PBE) in a simultaneous entry game consists of positioning policies $x^1, x^2 \in \{L, R\}$, advertising policies $a^2(x_1, x_2, \theta) \in \{P_2, N_1, \varnothing\}$, $a^1(x_1, x_2, \theta) \in \{P_1, N_2, \varnothing\}$ of firms, beliefs of consumers over firms types $\mathscr{F} : \theta_1 \times \theta_2 \rightarrow [0, 1]$, and purchase policies of consumers $\{g^j(x_1, x_2, a_1, a_2)\}_j \in \{1, 2, \varnothing\}$ such that

1. Consumers' choices are sequentially rational, i.e., $\{g^j(.)\}_j$ maximizes $E[U_{ij}|\mathscr{F}]$.
2. $a^2(.)$ and $a^1(.)$ constitute a Nash equilibrium of the advertising sub-game, given $\{g^j(.)\}_j$.
3. The location choices $x^1(.), x^2(.) \in \{L, R\}$ constitute a Nash equilibrium given $a^2(.)$, $a^1(.)$, and $\{g^j(.)\}_j$.
4. The consumers' beliefs $\mathscr{F}$ are updated based on $a^2(.)$ and $a^1(.)$ according to the Bayes' Rule.

When two firms simultaneously enter the market, the second stage of the game where firms determine their advertising is identical to the one in the benchmark model. Therefore, the lemmas in Section 3.2.1 still apply in the backward induction solution. What is different is the first-stage decisions about positioning. Yet, it turns out that the equilibria of the simultaneous entry game are similar to the equilibria of the entrant-incumbent game, with the slight modification that firms can choose to locate both on the right and on the left end of consumer heterogeneity line:

Proposition A.1. (Positioning under Simultaneous Entry)
(i) For each PBE of the entrant-incumbent game under co-location, there are two equivalent PBE in the simultaneous entry game, where both firms choose to locate at either R or L.
(ii) For each PBE of the entrant-incumbent game with location differentiation, there are two equivalent PBE in the simultaneous entry game, where one firm locates at R and other one firm locates at L.

Proposition A.1 demonstrates that the key insights of the benchmark model are robust to simultaneous entry of firms in a market. The equilibrium outcomes under a simultaneous game map to those under a sequential game.

A.2 Proofs of Propositions, Lemmas, and Corollaries

Proof of Proposition 1. The PBE given in Proposition 1 is defined by the following strategies and beliefs:

1. $x^2 = R$
2. $a^i(x_i, \theta) = \begin{cases} P_i, \text{if } P_i = \Pi \\ \varnothing, \text{if } P_i = 0 \end{cases}$
3. $\mathscr{F}$ puts probability 1 on $P_i = \Pi$ if $a_i = P_i$, 0 otherwise.
4. Let $B_{ij} = A_i - |\chi_j - x_i|$. If $\gamma_j < \max_i E[B_{ij}]$ then $g^j(x_2, a_1, a_2) = \varnothing$. If $\gamma_j \geq \max_i E[B_{ij}]$ and $\arg\max_i E[B_{ij}]$ is unique, then $g^j(x_2, a_1, a_2) = \arg\max_i E[B_{ij}]$. Otherwise,

$$g^j(x_2 = L, a_1, a_2) = \begin{cases} 1, \text{w.p. } 0.5 \\ 2, \text{w.p. } 0.5 \end{cases}$$

and

$$g^j(x_2 = R, a_1, a_2) = \begin{cases} 1, \text{ if } \chi_j = L \\ 2, \text{ if } \chi_j = R \end{cases} \quad .$$

We show that $\{x^2, a^1, a^2, \mathscr{F}, g^j\}$ is the unique PBE.

First, by definition, there exists a single g^j that maximizes $E[U_{ij}]$ for a given $\mathscr{F}$.

Second, note that the $\mathscr{F}$ is consistent with the advertising strategies of the firm types where $\mathscr{F}$, $E[A_i|a_i = P_i, a_{-i}, x_2] = (1 - \sigma_\Pi)\Pi$ and $E[A_i|a_i = \varnothing, a_{-i}, x_2] = -\sigma_\Pi \Pi$. See that the equilibrium is a separating one, i.e., each type follows a different advertising strategy. For a pooling equilibrium to exist, both types would have to announce $\varnothing$ because advertising has to be truthful. For there be no profitable deviations for the type with $P_i = \Pi$, consumer beliefs would have to put a probability less than 1 for $P_i = \Pi$ when P_i is announced which would fail the truthfulness assumption. Hence, $\mathscr{F}$ describes the unique beliefs in any PBE.

Third, in the advertising sub-game, given $\mathscr{F}$, announcing $a^i = P_i$ if $P_i = \Pi$ is strictly dominant, i.e., returns to running positive advertising is strictly larger than running no advertising:

$$E[A_i|a_i = P_i, a_{-i}, x_2] = (1 - \sigma_\Pi)\Pi > -\sigma_\Pi \Pi = E[A_i|a_i = \varnothing, a_{-i}, x_2], \quad \forall a_{-i}, x_2$$

Hence, a^1, a^2 is the unique Nash Equilibrium of the advertising sub-game given $\mathscr{F}$ and g^j.

Last, to prove the optimality of x^2, consider the potential outcomes following each location choice, summarized in Table A3.[32] The table restricts attention to the realizations of P_i. The realizations of N_i are irrelevant for firms' payoff because they cannot be advertised.

[32]Table A3 assumes $\Pi > \delta$. The proof is similar for $\Pi < \delta$.

θ	Prob.	a_1	a_2	$E[A_1]$	$E[A_2]$	D_2
$P_1 = P_2 = \Pi$	$\sigma_\Pi^2 + \rho\sigma_\Pi(1-\sigma_\Pi)$	P_1	P_2	$(1-\sigma_\Pi)\Pi$	$(1-\sigma_\Pi)\Pi$	$0.5\big[2 - \Gamma(-(1-\sigma_\Pi)\Pi) - \Gamma(\delta-(1-\sigma_\Pi)\Pi)\big]$
$P_1 = \Pi, P_2 = 0$	$\sigma_\Pi(1-\sigma_\Pi)(1-\rho)$	P_1	$\varnothing$	$(1-\sigma_\Pi)\Pi$	$-\sigma_\Pi\Pi$	0
$P_1 = 0, P_2 = \Pi$	$\sigma_\Pi(1-\sigma_\Pi)(1-\rho)$	$\varnothing$	P_2	$-\sigma_\Pi\Pi$	$(1-\sigma_\Pi)\Pi$	$2 - \Gamma(-(1-\sigma_\Pi)\Pi) - \Gamma(\delta-(1-\sigma_\Pi)\Pi)$
$P_1 = P_2 = 0$	$(1-\sigma_\Pi)(1-\sigma_\Pi+\rho\sigma_\Pi)$	$\varnothing$	$\varnothing$	$-\sigma_\Pi\Pi$	$-\sigma_\Pi\Pi$	$0.5\big[2 - \Gamma(\sigma_\Pi\Pi) - \Gamma(\delta+\sigma_\Pi\Pi)\big]$

Table A1: Co-Location

θ	Prob.	a_1	a_2	$E[A_1]$	$E[A_2]$	D_2
$P_1 = P_2 = \Pi$	σ_Π^2	P_1	P_2	$(1-\sigma_\Pi)\Pi$	$(1-\sigma_\Pi)\Pi$	$1 - \Gamma(-(1-\sigma_\Pi)\Pi)$
$P_1 = \Pi, P_2 = 0$	$\sigma_\Pi(1-\sigma_\Pi)$	P_1	$\varnothing$	$(1-\sigma_\Pi)\Pi$	$-\sigma_\Pi\Pi$	0
$P_1 = 0, P_2 = \Pi$	$\sigma_\Pi(1-\sigma_\Pi)$	$\varnothing$	P_2	$-\sigma_\Pi\Pi$	$(1-\sigma_\Pi)\Pi$	$2 - \Gamma(-(1-\sigma_\Pi)\Pi) - \Gamma(\delta-(1-\sigma_\Pi)\Pi)$
$P_1 = P_2 = 0$	$(1-\sigma_\Pi)^2$	$\varnothing$	$\varnothing$	$-\sigma_\Pi\Pi$	$-\sigma_\Pi\Pi$	$1 - \Gamma(\sigma_\Pi\Pi)$

Table A2: Locating Apart

Table A3: Demand for the Entrant with Negative Advertising Forbidden

We can show the expected payoff under locating apart exceeds the expected payoff under co-location in two steps. Assume the probabilities of realizations were identical across location choices. The payoff is identical for the asymmetric realizations of P_i. For the symmetric realizations, the payoff would have been larger under locating apart since

$$2 - 2\Gamma(-(1-\sigma_\Pi)\Pi) > 2 - \Gamma(-(1-\sigma_\Pi)\Pi) - \Gamma(\delta-(1-\sigma_\Pi)\Pi)$$

and

$$2 - 2\Gamma(\sigma_\Pi\Pi) > 2 - \Gamma(\sigma_\Pi\Pi) - \Gamma(\delta+\sigma_\Pi\Pi).$$

The extra term co-location has due to the difference in probabilities is

$$\frac{\rho\sigma_\Pi(1-\sigma_\Pi)}{2}\Big[\Gamma(-(1-\sigma_\Pi)\Pi) + \Gamma(\delta-(1-\sigma_\Pi)\Pi) - \Gamma(\sigma_\Pi\Pi) - \Gamma(\delta+\sigma_\Pi\Pi)\Big]$$

which is always negative. Hence, the expected payoff is necessarily higher under locating apart.

□

Proofs of Proposition 2, Lemmas 1, 2, and 4. The PBE is defined by

1. $x^2 = L$

2.
$$a^i(x_i = R, \theta) = \begin{cases} P_i, \text{ if } N_i = 0, P_{-i} = 0, P_i = \Pi \text{ (weak opponent)} \\ N_{-i}, \text{if above condition fails and } N_{-i} = -\beta \\ P_i, \text{ if above conditions fail and } P_i = \Pi \\ \varnothing, \text{otherwise} \end{cases}$$

and

$$a^i(x_i = L, \theta) = \begin{cases} P_i, \text{if } P_i = \Pi \\ N_{-i}, \text{if above condition fails and } N_{-i} = -\beta \\ \varnothing, \text{otherwise} \end{cases}$$

3. $\mathscr{F}$ can be constructed from Table 1.

4. Let $B_{ij} = A_i - |\chi_j - x_i|$
   - if $\gamma_j < \max_i E[B_{ij}]$ then $g^j(x_2, a_1, a_2) = \varnothing$
   - if above condition fails, $\arg\max_i E[B_{ij}]$ is unique, then $g^j(x_2, a_1, a_2) = \arg\max_i E[B_{ij}]$
   - otherwise, $g^j(x_2 = L, a_1, a_2) = \begin{cases} 1, \text{w.p. } 0.5 \\ 2, \text{w.p. } 0.5 \end{cases}$ and $g^j(x_2 = R, a_1, a_2) = \begin{cases} 1, \text{ if } \chi_j = L \\ 2, \text{ if } \chi_j = R \end{cases}$

We prove that $\{x^2, a^1, a^2, \mathscr{F}, g^j\}$ is a PBE when

(i) $\beta > (1 - \sigma_\Pi)\Pi + \delta$ (A1a)

(ii) $\bar{\gamma} + \sigma_\beta \beta \geq (1 - \sigma_\Pi)\Pi + \delta$ (A1b)

(iii) $\Pi > (1 - \rho)(1 - \sigma_\beta)\beta$ (A1c)

(iv) $\sigma_\beta(1 - \sigma_\beta)(1 - \sigma_\Pi)(1 - \rho)(1 - \rho + \rho\sigma_\Pi)\beta + (1 - \sigma_\Pi)\sigma_\Pi(1 - \rho)\Pi - \frac{\delta}{2} \geq \sigma_\beta(1 - \sigma_\beta)\beta$
$+ (1 - \sigma_\Pi)\sigma_\Pi(1 - \sigma_\beta)(1 - 3\sigma_\beta)\Pi - (1 - \sigma_\beta)(\sigma_\beta + (1 - \sigma_\beta)\sigma_\Pi(1 - \sigma_\Pi))\delta$ (A1d)

First, g^j, by definition, maximizes $E[U_{ij}]$ given $\mathscr{F}$.

Second, see that the $\mathscr{F}$ is consistent with the advertising of the firms. Importantly, when $P_i = \Pi$ and $N_{-i} = -\beta$, consumers expect firm i to use negative advertising against strong opponents under locating apart.

Third, we discuss the optimality of $a^i(.)$, and its uniqueness given $\mathscr{F}$. Please refer to Table 1 for the posterior beliefs following each advertising outcome.

Locating Apart: If $N_i = 0$ and $P_{-i} = 0$, i.e., the opponent is weak, then $E[A_i] > E[A_{-i}]$ regardless of whether firm i does positive or negative advertising. Then positive advertising would always lead to more demand except for the scenario where only negative advertising allows stealing consumers, that is,

$$E[A_i - A_{-i}|a_i = N_i, a_{-i} = \varnothing] > \delta \text{ and } E[A_i - A_{-i}|a_i = P_i, a_{-i} = \varnothing] \leq \delta,$$

which is ruled out by the other parameter inequalities and Assumption 1. Hence, prioritizing positive advertising against 'weak opponents' is strictly dominant under locating apart (Lemma 1). If either $N_i = -\beta$ or $P_{-i} = \Pi$, i.e., the opponent is strong, then, for negative advertising to be prioritized in the advertising equilibrium, negative advertising should be effective enough to steal consumers if the opponent runs positive advertising:

$$\begin{aligned} E[A_i - A_{-i}|a_i = N_i, a_{-i} = P_i] > \delta \\ \Leftrightarrow \beta - (1 - \sigma_\Pi)\Pi > \delta, \end{aligned} \tag{A2}$$

which is equivalent to condition (A1a) above. Second, for negative advertising to be prioritized against strong opponents in the unique advertising equilibrium, the number of stolen consumers should be sufficient to make up for lost demand in own location:

$$\begin{aligned} D_i(a_i = N_i, a_{-i} = P_i) \geq D_i(a_i = P_i, a_{-i} = P_i) \\ \Leftrightarrow 2 - \Gamma(-\sigma_\beta\beta) - \Gamma(\delta - \sigma_\beta\beta) \geq 1 - \Gamma(-\sigma_\beta\beta - (1 - \sigma_\Pi)\Pi) \\ \Leftrightarrow \bar{\gamma} + \sigma_\beta\beta \geq (1 - \sigma_\Pi)\Pi + \delta, \end{aligned} \tag{A3}$$

which is equivalent to condition (A1b) above. Since the game is symmetric, conditions (A1a) and (A1b) together imply that prioritizing negative advertising against strong opponents is strictly dominant (Lemma 2).

Co-Locating: If the opponent is weak, then $E[A_i] > E[A_{-i}]$ regardless of whether firm i does positive or negative advertising. Hence, prioritizing positive advertising against 'weak opponents' is strictly dominant under co-locating (Lemma 3).

If the opponent is strong, for positive advertising to be prioritized in the unique advertising equilibrium, it must be effective enough to steal consumers if the opponent runs negative advertising:

$$\begin{aligned} E[A_i - A_{-i}|a_i = P_i, a_{-i} = N_i] > 0 \\ \Leftrightarrow \Pi > (1 - \rho)(1 - \sigma_\beta)\beta, \end{aligned} \tag{A4}$$

which is equivalent to condition (A1b). Since the game is symmetric, this condition by itself implies that prioritizing positive advertising against strong opponents is strictly dominant under co-location (Lemma 4).

Lastly, to prove the optimality of x^2, consider the potential outcomes following each location choice, summarized in Table A6. Let $\xi_1 \equiv \sigma_\Pi(1 - \sigma_\Pi)(1 - \rho)$ and $\xi_2 \equiv (1 - \sigma_\Pi)(1 - \sigma_\Pi + \rho\sigma_\Pi)$ for convenience.

θ	Probability	a_1	a_2	D_2
$\{\Pi,\Pi,,\}$	$\sigma_\Pi^2+\rho\sigma_\Pi(1-\sigma_\Pi)$	P_1	P_2	$0.5\big[2-\Gamma(-(1-\sigma_\Pi)\Pi)-\Gamma(\delta-(1-\sigma_\Pi)\Pi)\big]$
$\{\Pi,0,-\beta,\}$	$\xi_1\sigma_\beta$	P_1	N_1	0
$\{0,\Pi,,-\beta\}$	$\xi_1\sigma_\beta$	N_2	P_2	$2-\Gamma((1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi)-\Gamma(\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi)$
$\{\Pi,0,0,\}$	$\xi_1(1-\sigma_\beta)$	P_1	$\varnothing$	0
$\{0,\Pi,,0\}$	$\xi_1(1-\sigma_\beta)$	$\varnothing$	P_2	$2-\Gamma(-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi)-\Gamma(\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi)$
$\{0,0,-\beta,-\beta\}$	$\xi_2\big(\sigma_\beta^2+\rho\sigma_\beta(1-\sigma_\beta)\big)$	N_2	N_1	$0.5\big[2-\Gamma(\sigma_\Pi\Pi+(1-\sigma_\beta)\beta)-\Gamma(\delta+\sigma_\Pi\Pi+(1-\sigma_\beta)\beta)\big]$
$\{0,0,0,-\beta\}$	$\xi_2\big(\sigma_\beta(1-\sigma_\beta)(1-\rho)\big)$	N_2	$\varnothing$	0
$\{0,0,-\beta,0\}$	$\xi_2\big(\sigma_\beta(1-\sigma_\beta)(1-\rho)\big)$	$\varnothing$	N_1	$2-\Gamma(\sigma_\Pi\Pi-\sigma_\beta\beta)-\Gamma(\delta+\sigma_\Pi\Pi-\sigma_\beta\beta)$
$\{0,0,0,0\}$	$\xi_2(1-\sigma_\beta)(1-\sigma_\beta+\rho\sigma_\beta)$	$\varnothing$	$\varnothing$	$0.5\big[2-\Gamma(\sigma_\Pi\Pi-\sigma_\beta\beta)-\Gamma(\delta+\sigma_\Pi\Pi-\sigma_\beta\beta)\big]$

Table A4: Co-Location

θ	Probability	a_1	a_2	D_2
$\{,,-\beta,-\beta\}$	σ_β^2	N_2	N_1	$1-\Gamma(1-\sigma_\beta)\beta)$
$\{,\Pi,0,-\beta\}$	$\sigma_\beta(1-\sigma_\beta)\sigma_\Pi$	N_2	P_2	0
$\{\Pi,,-\beta,0\}$	$\sigma_\beta(1-\sigma_\beta)\sigma_\Pi$	P_1	N_1	$2-\Gamma(-\sigma_\beta\beta)-\Gamma(\delta-\sigma_\beta\beta)$
$\{,0,0,-\beta\}$	$\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)$	N_2	$\varnothing$	0
$\{0,,-\beta,0\}$	$\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)$	$\varnothing$	N_1	$2-\Gamma(\sigma_\Pi\Pi-\sigma_\beta\beta)-\Gamma(\delta+\sigma_\Pi\Pi-\sigma_\beta\beta)$
$\{\Pi,\Pi,0,0\}$	$(1-\sigma_\beta)^2\sigma_\Pi^2$	P_1	P_2	$1-\Gamma(-(1-\sigma_\Pi)\Pi-\sigma_\beta\beta)$
$\{\Pi,0,0,0\}$	$(1-\sigma_\beta)^2\sigma_\Pi(1-\sigma_\Pi)$	P_1	$\varnothing$	0
$\{0,\Pi,0,0\}$	$(1-\sigma_\beta)^2\sigma_\Pi(1-\sigma_\Pi)$	$\varnothing$	P_2	$2-\Gamma(-(1-\sigma_\Pi)\Pi-\sigma_\beta\beta)-\Gamma(\delta-(1-\sigma_\Pi)\Pi-\sigma_\beta\beta)$
$\{0,0,0,0\}$	$(1-\sigma_\beta)^2(1-\sigma_\Pi)^2$	$\varnothing$	$\varnothing$	$1-\Gamma(\sigma_\Pi\Pi-\sigma_\beta\beta)$

Table A5: Locating Apart

Table A6: Demand for the Entrant with Negative Advertising Allowed Note. D_2 denotes the demand for the entrant. See Table 1 for $E[A_1]$ and $E[A_2]$ associated with each outcome. $\theta=\{P_1,P_2,N_1,N_2\}$. If the associated entry in θ is unspecified, that means a_1, a_2, and D_2 do not depend on the value of that entry.

With some algebra, we can simplify to

$$E[D_2|x_2=L]=\frac{1}{\bar{\gamma}-\underline{\gamma}}\Big[\bar{\gamma}+\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)(1-\rho)(1-\rho+\rho\sigma_\Pi)\beta + (1-\sigma_\Pi)\sigma_\Pi(1-\rho)\Pi-\frac{\delta}{2}\Big],\text{ and} \tag{A5}$$

$$E[D_2|x_2=R]=\frac{1}{\bar{\gamma}-\underline{\gamma}}\Big[\bar{\gamma}+\sigma_\beta(1-\sigma_\beta)\beta+(1-\sigma_\Pi)\sigma_\Pi(1-\sigma_\beta)(1-3\sigma_\beta)\Pi - (1-\sigma_\beta)(\sigma_\beta+(1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\delta\Big]. \tag{A6}$$

Hence, $E[D_2|x_2=L]\geq E[D_2|x_2=R]$ becomes equivalent to condition (A1d) above.

To sum up, once conditions (A1a)-(A1d) are satisfied, there exists a PBE as defined in $1-4$, which is unique given the beliefs specified by $\mathscr{F}$. □

Proof of Corollary 1. To prove the corollary, first, notice that the only advertising equilibria that can sustain co-location are where co-location leads to positive and locating apart leads to negative advertising being prioritized. Second, see that conditions (A1a) and (A1b), which are necessary for the entrant to co-locate given $\mathscr{F}$ as proven in Proposition 2, are only satisfied when δ is small enough. Let δ_1 and δ_2 be the values of δ which make the lefthand sides equal to the righthand sides of (A1a) and (A1b), respectively. Second, the condition in (A1d) is necessary for the entrant to co-locate given $\mathscr{F}$. The term multiplying δ on the lefthand side is 0.5 while it is $(1-\sigma_\beta)(\sigma_\beta+(1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))$ on the righthand side. Since the maximum of $x(1-x)$ is 0.25 for $x<1$, the latter term is bounded above by 0.5. Hence, as δ grows, the term with δ on the lefthand side dominates the other terms and (A1d) fails. Let δ_3 be the value of δ which makes the lefthand side equal to the righthand side. Then, for $\bar{\delta}=\min\{\delta_1,\delta_2,\delta_3\}$, the corollary follows. □

Proof of Corollary 2. First, see that (A1c) is only satisfied when ρ is large enough. Let $\underline{\rho}$ be the value of ρ where the lefthand side equals righthand side. Second, in (A1d), see that the terms with β and Π on the lefthand side disappear as ρ approaches 1. Because the coefficient of δ on the righthand side is always smaller in magnitude relative to the coefficient of δ on the lefthand side (see the Proof of Corollary 1), (A1d) fails as ρ approaches 1. Let $\bar{\rho}$ be the largest value of ρ where the lefthand side equals righthand side.[33] The corollary follows. □

Proof of Corollary 3. Let the value of σ_β where the lefthand side equals righthand side in (A1c) be $\underline{\sigma_\beta}$. Similarly, let the values of σ_Π where the lefthand side equals righthand side in (A1a) and (A1b) be $\sigma_{\Pi,1}$ and $\sigma_{\Pi,2}$ respectively, conditional on $\sigma_\beta=\underline{\sigma_\beta}$. Then, let $\underline{\sigma_\Pi}=\max\{\sigma_{\Pi,1},\sigma_{\Pi,2}\}$. The corollary follows. The reader should be careful in constructing the proof; among σ_Π, σ_β, Π, and β, there are only three free parameters because of the normalization $\sigma_\Pi\Pi=\sigma_\beta\beta$. □

Proof of Proposition 3. Here, we prove that for any set of parameters where the equilibrium in Proposition 2 (denoted by CO_{PN}) exists, there exists another equilibrium (denoted by LA_{NN}) where prioritize negative advertising against strong opponents following each location choice, and the entrant locates apart. To that end, first, we characterize the parameter set where LA_{NN} exists. Second, we show that the parameter set where the CO_{PN} exists is a strict subset of the parameter set where LA_{NN} exists.

The beliefs that are consistent with the advertising strategies in LA_{NN} are as described in Table A7.

Given the beliefs described in Table A7, (1) the necessary and sufficient conditions for negative advertising to be prioritized against strong opponents under locating apart, and (2) the necessary and sufficient conditions for positive advertising to be prioritized against weak opponents are identical to those for CO_{PN} (i.e., (A1a) and (A1b)). This is because consumer beliefs regarding prioritized advertising are identical under these scenarios. The necessary and sufficient conditions for negative advertising to be prioritized against strong opponents under co-location are:[34]

[33]See that neither $\underline{\rho}$ nor $\bar{\rho}$ are necessarily between 0 and 1.
[34]These conditions follow from the same reasoning given for (A1a) and (A1b) in the Proof of Proposition 2.

Locating Apart

a_i	a_{-i}	Prioritized Advertising i	$-i$	$\tilde{\mathscr{F}}(P_i, N_i, P_{-i}, N_{-i})$	$E[A_i]$	$E[A_{-i}]$
P_i	P_{-i}	N_{-i}	N_i	$\{1,0,1,0\}$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$
P_i	N_i	N_{-i}	N_i	$\{1,1,\sigma_\Pi,0\}$	$(1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta$	$\sigma_\beta\beta$
N_{-i}	N_i	N_{-i}	N_i	$\{\sigma_\Pi,1,\sigma_\Pi,1\}$	$-(1-\sigma_\beta)\beta$	$-(1-\sigma_\beta)\beta$
P_i	$\varnothing$	P_i	N_i	$\{1,0,0,\sigma_\beta\}$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi$
N_{-i}	$\varnothing$	P_i	N_i	$\{0,0,0,1\}$	$\sigma_\beta\beta - \sigma_\Pi\Pi$	$-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$
$\varnothing$	$\varnothing$	P_i	P_{-i}	$\{0,0,0,0\}$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$

Co-location

a_i	a_{-i}	Prioritized Advertising i	$-i$	$\tilde{\mathscr{F}}(P_i, N_i, P_{-i}, N_{-i})$	$E[A_i]$	$E[A_{-i}]$
P_i	P_{-i}	N_{-i}	N_i	$\{1,0,1,0\}$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$
P_i	N_i	N_{-i}	N_i	$\{1,1,\sigma_\Pi,0\}$	$(1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta$	$\sigma_\beta\beta + \rho(1-\sigma_\Pi)\Pi$
N_{-i}	N_i	N_{-i}	N_i	$\{\sigma_\Pi,1,\sigma_\Pi,1\}$	$-(1-\sigma_\beta)\beta$	$-(1-\sigma_\beta)\beta$
P_i	$\varnothing$	P_i	N_i	$\{1,0,0,\sigma_\beta\}$	$(1-\sigma_\Pi)\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$
N_{-i}	$\varnothing$	P_i	N_i	$\{0,0,0,1\}$	$\sigma_\beta\beta - \sigma_\Pi\Pi$	$-\sigma_\Pi\Pi - (1-\sigma_\beta)\beta$
$\varnothing$	$\varnothing$	P_i	P_{-i}	$\{0,0,0,0\}$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$	$-\sigma_\Pi\Pi + \sigma_\beta\beta$

Table A7: Posterior Beliefs Given Advertising Outcomes Under LA_{NN}. Note: Terms in purple indicate the "direct effect" of advertising, in black refer to the "inference effect," and in blue refer to the "spillover effect."

$$\beta > (1-\rho)(1-\sigma_\Pi)\Pi \tag{A7}$$

$$\bar{\gamma} + \sigma_\beta\beta > \frac{\delta}{2} - (2\rho - 1)(1-\sigma_\Pi)\Pi \tag{A8}$$

Notice any set of parameters that satisfy (A7) will also satisfy (A1a) and any set of parameters that satisfy (A8) will also satisfy (A1b). In other words, when consumers believe negative advertising is prioritized, whenever firms find prioritizing negative advertising to be profitable under locating apart, they will also find it profitable under co-location. Furthermore, the entrant will locate apart whenever the advertising outcomes are identical following each location choice. So, for LA_{NN} to exist, (A1a) and (A1b) are sufficient conditions. For CO_{PN} to exist, however, (A1c) and (A1d) also need to be satisfied. Because (A1c) and (A1d) are distinct from (A1a) and (A1b), the set of parameters where CO_{PN} exists is a strict subset of the set of parameters where LA_{NN} exists. Therefore, there are no set of parameters where CO_{PN} is the unique equilibrium.

□

Proof of Lemma 6. We start by deriving a simple expression for welfare comparisons. Since the outside option provides 0 utility, the expected total consumer surplus can be written as

$$CS = \sum_{\theta} P(\theta)\Big[\int_{\gamma_L^*(\theta)}^{\bar{\gamma}} (\gamma - \gamma_L^{true}(\theta))d\Gamma(\gamma) + \int_{\gamma_R^*(\theta)}^{\bar{\gamma}} (\gamma - \gamma_R^{true}(\theta))d\Gamma(\gamma)\Big], \tag{A9}$$

where θ refers to the vector of values for product attributes, and $\gamma^*(\theta)$ and $\gamma^{true}(\theta)$ denote the reservation values for the consumers who are indifferent between buying the superior product or the outside option ex-ante and ex-post, respectively. The two values can differ because advertising does not always reveal all attributes of the product. If $\gamma^* > \gamma^{true}$, there are some consumers who don't buy a product, but would have enjoyed a positive utility and when $\gamma^* < \gamma^{true}$, there are some consumers who buy a product, but would have been better off with the outside option. When Γ is the uniform *cdf*, the expression becomes

$$\begin{aligned}
CS &= \sum_{\theta} P(\theta)\Big[\int_{\gamma_L^*(\theta)}^{\bar{\gamma}} \frac{\gamma - \gamma_L^{true}(\theta)}{\bar{\gamma} - \underline{\gamma}} d\gamma \int_{\gamma_R^*(\theta)}^{\bar{\gamma}} \frac{\gamma - \gamma_R^{true}(\theta)}{\bar{\gamma} - \underline{\gamma}} d\gamma\Big] \\
&= \sum_{\theta} P(\theta)\Big[\frac{(\bar{\gamma} - \gamma_L^*(\theta))(\bar{\gamma} + \gamma_L^*(\theta) - 2\gamma_L^{true}(\theta))}{2(\bar{\gamma} - \underline{\gamma})} + \frac{(\bar{\gamma} - \gamma_R^*(\theta))(\bar{\gamma} + \gamma_R^*(\theta) - 2\gamma_R^{true}(\theta))}{2(\bar{\gamma} - \underline{\gamma})}\Big] \\
&= \sum_{\theta} P(\theta)\Big[\Phi + \gamma_L^*(\theta)(2\gamma_L^{true}(\theta) - \gamma_L^*(\theta)) - 2\bar{\gamma}\gamma_L^{true}(\theta) \\
&\qquad + \gamma_R^*(\theta)(2\gamma_R^{true}(\theta) - \gamma_R^*(\theta)) - 2\bar{\gamma}\gamma_R^{true}(\theta)\Big],
\end{aligned} \tag{A10}$$

where Φ is only a function of $\bar{\gamma}$ and $\underline{\gamma}$, hence, it is invariant to advertising policy. Using the equilibrium strategies of firms and consumers in Propositions 1 and 2, we can characterize the values of γ^* and γ^{true} for each realization of product attributes (See Table A10).

θ	a_1	a_2	W	γ_L^*	γ_R^*	γ_L^{true}	γ_R^{true}
$\{\Pi,\Pi,\beta,\beta\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,\Pi,\beta,0\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,\Pi,0,\beta\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,\Pi,0,0\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,\beta,\beta\}$	P	0	1	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,\beta,0\}$	P	0	1	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,0,\beta\}$	P	0	1	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,0,0\}$	P	0	1	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,\beta,\beta\}$	0	P	2	$\delta-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,\beta,0\}$	0	P	2	$\delta-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,0,\beta\}$	0	P	2	$\delta-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,0,0\}$	0	P	2	$\delta-(1-\sigma_\Pi)\Pi$	$-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{0,0,\beta,\beta\}$	0	0	-	$\sigma_\Pi\Pi$	$\sigma_\Pi\Pi$	$(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$	$(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$
$\{0,0,\beta,0\}$	0	0	-	$\sigma_\Pi\Pi$	$\sigma_\Pi\Pi$	$(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$	$\sigma_\Pi\Pi-\sigma_\beta\beta$
$\{0,0,0,\beta\}$	0	0	-	$\sigma_\Pi\Pi$	$\sigma_\Pi\Pi$	$\sigma_\Pi\Pi-\sigma_\beta\beta$	$(1-\sigma_\beta)\beta+\sigma_\Pi\Pi$
$\{0,0,0,0\}$	0	0	-	$\sigma_\Pi\Pi$	$\sigma_\Pi\Pi$	$\sigma_\Pi\ \Pi-\sigma_\beta\beta$	$\sigma_\Pi\Pi-\sigma_\beta\beta$

Table A8: Negative Advertising Banned, Firms are Located Apart

θ	a_1	a_2	W	γ_L^*	γ_R^*	γ_L^{true}	γ_R^{true}
$\{\Pi,\Pi,\beta,\beta\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,\Pi,\beta,0\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,\Pi,0,\beta\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(0.5-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,\Pi,0,0\}$	P	P	-	$-(1-\sigma_\Pi)\Pi$	$\delta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,\beta,\beta\}$	P	N	1	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,\beta,0\}$	P	N	1	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,0,\beta\}$	P	0	1	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{\Pi,0,0,0\}$	P	0	1	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,\beta,\beta\}$	N	P	2	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,\beta,0\}$	0	P	2	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,0,\beta\}$	N	P	2	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$	$\delta+(1-\sigma_\beta)\beta-(1-\sigma_\Pi)\Pi$
$\{0,\Pi,0,0\}$	0	P	2	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$	$\delta-\sigma_\beta\beta-(1-\sigma_\Pi)\Pi$
$\{0,0,\beta,\beta\}$	N	N	-	$\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$	$\delta+\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$	$\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$	$\delta+\sigma_\Pi\Pi+(1-\sigma_\beta)\beta$
$\{0,0,\beta,0\}$	0	N	2	$\sigma_\Pi\Pi-\sigma_\beta\beta$	$\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$	$\sigma_\Pi\Pi-\sigma_\beta\beta$	$\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$
$\{0,0,0,\beta\}$	N	0	1	$\sigma_\Pi\Pi-\sigma_\beta\beta$	$\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$	$\sigma_\Pi\Pi-\sigma_\beta\beta$	$\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$
$\{0,0,0,0\}$	0	0	-	$\sigma_\Pi\Pi-\sigma_\beta\beta$	$\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$	$\sigma_\Pi\Pi-\sigma_\beta\beta$	$\delta+\sigma_\Pi\Pi-\sigma_\beta\beta$

Table A9: Negative Advertising Allowed, Firms are Co-located

Table A10: The γ values for which Ex-ante and Ex-post Utilities are 0, See Table 1 for $E[A_1]$ and $E[A_2]$ associated with each outcome. $\theta = \{P_1, P_2, N_1, N_2\}$. a_1 and a_2 are the advertising outcomes that arise in the equilibria described in Propositions 1 and 2. W refers to which firm's product (if any) serves the whole market.

Given the values for γ^* and γ^{true}, we can characterize the consumer surplus associated with each realization of product attributes. In Table A11, we tabulate the associated Consumer Surplus separately for consumers located in L and R. We omit the Φ term in (A10), since it does not vary across different scenarios. Let $\xi_3 = (1 - \sigma_\Pi)\Pi$ for convenience.

	Negative Advertising Banned	Negative Advertising Allowed	
θ	Consumer Surplus (L)	Consumer Surplus (L)	Consumer Surplus (R)
$\{\Pi,\Pi,\beta,\beta\}$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{\Pi,\Pi,\beta,0\}$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$	$\xi_3(\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(0.5-\sigma_\beta)\beta$	$*-\delta(2\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{\Pi,\Pi,0,\beta\}$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$	$\xi_3(\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(0.5-\sigma_\beta)\beta$	$*-\delta(2\xi_3-2(0.5-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{\Pi,\Pi,0,0\}$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$	$*-\delta(2\xi_3+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$
$\{\Pi,0,\beta,\beta\}$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{\Pi,0,\beta,0\}$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{\Pi,0,0,\beta\}$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{\Pi,0,0,0\}$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{0,\Pi,\beta,\beta\}$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-2\delta)+2(1-\sigma_\beta)\beta(\delta-\bar{\sigma})-2\bar{\gamma}\delta+\delta^2$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{0,\Pi,\beta,0\}$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}-2\delta)+2\sigma_\beta\beta(\bar{\sigma}-\delta)-2\bar{\gamma}\delta+\delta^2$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{0,\Pi,0,\beta\}$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-2\delta)+2(1-\sigma_\beta)\beta(\delta-\bar{\sigma})-2\bar{\gamma}\delta+\delta^2$	$\xi_3(\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}))+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{0,\Pi,0,0\}$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}-2\delta)+2\sigma_\beta\beta(\bar{\sigma}-\delta)-2\bar{\gamma}\delta+\delta^2$	$\xi_3(\xi_3+2\sigma_\beta\beta+2\bar{\gamma}))+\sigma_\beta\beta(-\sigma_\beta\beta+2\bar{\gamma})$	$*-\delta(2\xi_3-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{0,0,\beta,\beta\}$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi+2(1-\sigma_\beta)\beta-2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi+2(1-\sigma_\beta)\beta-2\bar{\gamma})+(1-\sigma_\beta)\beta((1-\sigma_\beta)\beta-2\bar{\gamma})$	*-$\delta(-2\sigma_\Pi\Pi-2(1-\sigma_\beta)\beta+2\bar{\gamma}-\delta)$
$\{0,0,\beta,0\}$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi+2(1-\sigma_\beta)\beta-2\bar{\gamma})-2\bar{\gamma}(1-\sigma_\beta)\beta$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+\sigma_\beta\beta(\sigma_\beta\beta+2\bar{\gamma})$	*-$\delta(-2\sigma_\Pi\Pi+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$
$\{0,0,0,\beta\}$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+\sigma_\beta\beta(\sigma_\beta\beta+2\bar{\gamma})$	*-$\delta(-2\sigma_\Pi\Pi+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$
$\{0,0,0,0\}$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+2\bar{\gamma}\sigma_\beta\beta$	$\sigma_\Pi\Pi(\sigma_\Pi\Pi-2\sigma_\beta\beta-2\bar{\gamma})+\sigma_\beta\beta(\sigma_\beta\beta+2\bar{\gamma})$	*-$\delta(-2\sigma_\Pi\Pi+2\sigma_\beta\beta+2\bar{\gamma}-\delta)$

Table A11: The consumer surplus following the equilibrium described in Proposition 2 **Notes:** (L) and (R) refer to the consumer surplus associated with consumers in locations L and R, respectively. The values are suppressed for consumers in R for when negative advertising is not permitted, as they are symmetrical to those for consumers in L. $*$ refers to the Consumer Surplus value for the consumers in location L in the same row, so the expression following $*$ gives the additional surplus consumers located at R enjoy relative to consumers located at L when negative advertising is allowed.

When both products have the positive attribute, both firms utilize positive advertising in equilibrium regardless of whether negative advertising is allowed and regardless of whether the products have the negative attribute or not. Hence, consumers can conclude the positive attribute is present in both products with probability 1, yet gain no additional information about the presence of the negative attribute. This is reflected in the first four rows of Table A11 which show that when $P_1 = P_2 = \Pi$, the consumer welfare becomes identical between the two scenarios as $\delta \longrightarrow 0$.

If one or more products lacks the positive attribute, the firm(s) that lacks the positive attribute (say firm i) will not be able to run positive advertising. Because positive advertising is prioritized, consumers can conclude that the positive attribute is missing in firm i's product with probability 1, regardless of whether negative advertising is allowed. If negative advertising is forbidden, consumers do not learn anything else, because firm i has to run no advertising regardless of the presence of negative attributes. If negative advertising is allowed, however, consumers also learn whether firm $-i$ has the negative attribute. If firm i runs negative advertising, consumers can conclude the negative attribute is present in firm $-i$'s product with probability 1. If firm i runs no advertising, consumers can conclude that the negative attribute is present in firm $-i$'s product with probability 0. This is reflected in the additional β terms in rows 5-16 when negative advertising is permitted in Table A11.

To tease out the change in surplus through changes in the available information, we take two steps. First, we compute the change in expected surplus when negative advertising is permitted as if the probability distribution of θ is the same for both and equal to the one under locating apart (i.e. $\rho = 0$). Second, we take $\delta \longrightarrow 0$ to suppress the change in consumer surplus due to reduced product diversity. The resulting term can be simplified as

$$4\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)\beta\big(2\sigma_\Pi\beta + (1-\sigma_\Pi)\bar{\gamma}\big), \tag{A11}$$

which is always positive. Hence, we conclude that the consumers have welfare gains due to additional information when negative advertising is permitted.

Lastly, the variance of β equals $\sigma_\beta(1-\sigma_\beta)$ as it has a Bernoulli distribution. See that both this variance and (A11) approach 0 as σ_β approaches 0 or 1. Hence, the welfare gains due to additional information disappear as the variance of β goes to 0. □

Proof of Lemma 7. The welfare loss due to differentiation is reflected in the additional δ terms when negative advertising is permitted in Table A11. When one firm serves the whole market, there are no additional δ terms, because consumers in one of the locations end up purchasing from a firm in the other location, regardless of whether negative advertising is permitted or not (rows 5-12). In the other outcomes (rows 1-4 and 13-16), however, there are additional terms with δ when negative advertising is permitted. The presence of such terms indicate that some consumers who could buy a product that exactly matches their preference when negative advertising is not allowed can only buy from a firm that doesn't match their preference when negative advertising is permitted. This is because when negative advertising is permitted, the firms co-locate at L, hence, consumers located at R will necessarily buy from the other location.

To tease out the change in surplus through changes in the product differentiation, we take two steps. First, we compute the change in expected surplus when negative advertising is permitted as if the probability distribution of θ is the same for both and equal to the one under locating apart (i.e. $\rho = 0$). Second, we isolate the terms with δ.[35] The resulting term can be simplified as:

$$\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(1-2\sigma_\Pi)\Pi - 2\sigma_\beta(1-\sigma_\Pi)^2(1-\sigma_\beta)\beta - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\bar{\gamma}-\delta)\Big]. \tag{A12}$$

Next, we prove the term in (A12) is always negative, i.e., there is some welfare *loss* associated with the change in product differentiation. Using the normalization made earlier, plugging in $\sigma_\Pi\Pi = \sigma_\beta\beta$ yields:

$$\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(\sigma_\beta - \sigma_\Pi - \sigma_\beta\sigma_\Pi)\Pi - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\bar{\gamma}-\delta)\Big]. \tag{A13}$$

Replacing $\bar{\gamma}$ in (A13) with $\delta + \sigma_\Pi\Pi + \sigma_\beta\beta$ yields:

$$\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(\sigma_\beta - \sigma_\Pi - \sigma_\beta\sigma_\Pi)\Pi - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\sigma_\Pi\Pi + 2\sigma_\beta\beta + \delta)\Big]. \tag{A14}$$

From Assumption 1, $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$. Hence, proving (A14) is negative is sufficient to prove (A13) is negative. Next, notice that the term on the left, which is the only positive term in the expression, is maximized when $\sigma_\beta = 1$. Setting $\sigma_\beta = 1$ and collecting the terms with Π gives

$$\delta\Big[-2\sigma_\Pi^2\Pi - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\sigma_\beta\beta + \delta)\Big], \tag{A15}$$

which is necessarily negative. Then (A12) has to be negative. Hence, consumers have welfare losses due to reduced product differentiation when negative advertising is permitted.

Lastly, to prove that welfare losses increase as δ increases, we take the derivative of (A12) with respect to δ, which yields

$$(2\delta - 2\bar{\gamma})(\sigma_\Pi^2 + (1-\sigma_\Pi)^2) + 2\sigma_\Pi(1-\sigma_\Pi)(\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi)\Pi. \tag{A16}$$

Notice that $\sigma_\Pi^2 + (1-\sigma_\Pi)^2 > (1-\sigma_\Pi)(\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi)$ for any value of σ_Π and σ_β. Since $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$ by Assumption 1, the derivative term is necessarily negative. Therefore, the welfare loss due to reduced product differentiation increases with higher δ. □

[35] In other words, we remove the terms that are associated with additional information, given in the Proof of Lemma 6.

Proof of Proposition 4. Using Table A11, we can write down the change in expected consumer surplus when negative advertising is permitted:

$$\Delta CS = \underbrace{4\sigma_\beta(1-\sigma_\beta)(1-\sigma_\Pi)\beta\big(2\sigma_\Pi\beta + (1-\sigma_\Pi)\bar{\gamma}\big)}_{\text{Information Gains}} \tag{A17}$$
$$+ \underbrace{\delta\Big[2\sigma_\Pi(1-\sigma_\Pi)(1-2\sigma_\Pi)\Pi - 2\sigma_\beta(1-\sigma_\Pi)^2(1-\sigma_\beta)\beta - (\sigma_\Pi^2 + (1-\sigma_\Pi)^2)(2\bar{\gamma}-\delta)\Big]}_{\text{Product Differentiation Losses}}$$
$$+ 2\rho(1-\sigma_\Pi)\Big[\sigma_\Pi(2\sigma_\Pi - 1)\Pi(\Pi+\delta) - 2\sigma_\Pi\Pi\bar{\gamma} - \sigma_\Pi\sigma_\beta(1-\sigma_\beta)\beta^2$$
$$+ \sigma_\beta(1-\sigma_\beta)(2\sigma_\Pi - \rho\sigma_\Pi - 1)\beta(2\bar{\gamma} - \beta - \delta)\Big].$$

The first and second terms denote the welfare change due to information and reduced product differentiation that were derived in Lemmas 6 and 7, respectively. These terms were derived keeping the distribution of θ fixed when negative advertising was permitted. The third term is the 'correction' term, which reflects the change in welfare due to the change in the distribution of θ and disappears as $\rho \longrightarrow 0$.

Collecting the terms, and using the normalization $\sigma_\Pi\Pi = \sigma_\beta\beta$, we can re-write (A17) as the summation of two terms:

$$\begin{aligned}
\Delta CS = {} & 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[4\sigma_\Pi(1-\sigma_\beta)\beta + 2(1-\sigma_\Pi)(1-\sigma_\beta)\bar{\gamma} + (\sigma_\beta - \sigma_\Pi - \rho\sigma_\Pi)\delta \\
& + \rho(2\sigma_\Pi - 1)\Pi + \rho(2\sigma_\Pi - 1)\delta - 2\rho\bar{\gamma} - \rho\sigma_\Pi(1-\sigma_\beta)\beta \\
& + \rho(1-\sigma_\beta)(2\sigma_\Pi - 1 - \rho\sigma_\Pi)(2\bar{\gamma} - \beta - \delta)\Big] \\
& - \delta(2\bar{\gamma} - \delta)\Big[\sigma_\Pi^2 + (1-\sigma_\Pi)^2\Big] \\
= {} & 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[(1-\sigma_\beta)\big(\sigma_\Pi(4 - 3\rho + \rho^2) + \rho\big)\beta \\
& + 2\big((1-\sigma_\beta)(1 - \sigma_\Pi + \rho(2-\rho)\sigma_\Pi - \rho) - \rho\big)\bar{\gamma} \\
& + \big(\sigma_\beta(1-\sigma_\Pi) - \sigma_\Pi(1-\rho^2) + \sigma_\beta\sigma_\Pi(2\rho - rho^2 - 1)\big)\delta \\
& + \rho(2\sigma_\Pi - 1)\Pi\Big] \\
& - \delta(2\bar{\gamma} - \delta)\Big[\sigma_\Pi^2 + (1-\sigma_\Pi)^2\Big].
\end{aligned} \tag{A18}$$

Notice that ΔCS is continuous in all parameters. The second term is necessarily negative due to Assumption 1. The first term in (A18), whose sign cannot be established with certainty

- disappears if σ_Π approaches 0 or 1, or σ_β approaches 0, and
- becomes negative as σ_β approaches 1.

It is straightforward how the first term in (A18) disappears as σ_Π approaches 0 or 1. Notice that, σ_β approaching 0 is equivalent to σ_Π approaching 0, given the normalization $\sigma_\beta\beta = \sigma_\Pi\Pi$.[36] Lastly, to show that the term becomes negative as σ_β approaches 1, we plug $\sigma_\beta = 1$ in (A18), which yields

$$\begin{aligned}\Delta CS = 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[0\beta - 2\rho\bar{\gamma} + (1-3\sigma_\Pi+2\rho\sigma_\Pi)\delta + \rho(2\sigma_\Pi-1)\Pi\Big] \\ -\delta(2\bar{\gamma}-\delta)\Big[\sigma_\Pi^2 + (1-\sigma_\Pi)^2\Big].\end{aligned} \tag{A19}$$

The term in the brackets is strictly negative because (1) $1-3\sigma_\Pi+2\rho\sigma_\Pi$ is bounded above by 1, (2) $\rho(2\sigma_\Pi-1)$ is bounded above by σ_Π and (3) $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$ by Assumption 1. Moreover, ΔCS will be negative for any $\sigma_\beta > \underline{\sigma_\beta}$ for some $\underline{\sigma_\beta} < 1$ because ΔCS is continuous in $\sigma_\beta\beta$.

Lastly, we prove that the change in welfare decreases with δ. Notice that the first term in (A17) does not change with δ while the second term decreases as shown in Lemma 7. Taking the derivative of (A17) with respect to δ and using $\sigma_\Pi\Pi = \sigma_\beta\beta$ yields

$$2(\delta-\bar{\gamma})(\sigma_\Pi^2+(1-\sigma_\Pi)^2) + 2\sigma_\Pi(1-\sigma_\Pi)\Pi\Big[(\sigma_\beta(1-\sigma_\Pi)-\sigma_\Pi) + \rho\Big((2\sigma_\Pi-1)-(1-\sigma_\beta)(2\sigma_\Pi-\rho\sigma_\Pi-1)\Big)\Big]. \tag{A20}$$

By Assumption 1, $\bar{\gamma} > \delta + \sigma_\Pi\Pi + \sigma_\beta\beta$, so a sufficient condition for the term to be negative is:

$$\begin{aligned}(1-\sigma_\Pi)\Big[\sigma_\beta(1-\sigma_\Pi)-\sigma_\Pi+\rho\Big((2\sigma_\Pi-1)-(1-\sigma_\beta)(2\sigma_\Pi-\rho\sigma_\Pi-1)\Big)\Big] \\ < (\sigma_\Pi^2+(1-\sigma_\Pi)^2).\end{aligned} \tag{A21}$$

See that the term in brackets is bounded above by $1-\sigma_\Pi$:

$$\begin{aligned}&\sigma_\beta(1-\sigma_\Pi)-\sigma_\Pi+\rho\Big((2\sigma_\Pi-1)-(1-\sigma_\beta)(2\sigma_\Pi-\rho\sigma_\Pi-1)\Big) \\ &= \sigma_\beta(1-\sigma_\Pi)-\sigma_\Pi+\rho\Big((1-\sigma_\beta)\rho\sigma_\Pi+\sigma_\beta\sigma_\Pi-\sigma_\beta(1-\sigma_\Pi))\Big) \\ &= \sigma_\beta(1-\rho)(1-\sigma_\Pi)-\sigma_\Pi+\rho\Big((1-\sigma_\beta)\rho\sigma_\Pi+\sigma_\beta\sigma_\Pi)\Big) \\ &< \sigma_\beta(1-\rho)(1-\sigma_\Pi)-\sigma_\Pi+\sigma_\Pi \\ &< 1-\sigma_\Pi.\end{aligned}$$

Thus, the condition is satisfied for $\sigma_\Pi > 0$ and the total change in consumer surplus given in (A17) decreases with δ. □

Proof of Lemma 8. We postulate equilibrium strategies for the pricing search sub-games after

[36] A similar argument cannot be made for Π and β because δ is bounded above by the two in the equilibrium parameter set, hence, the second term disappears together with the first term.

each location and advertising choices, and prove that the postulated strategies indeed constitute a Nash equilibrium.

If $x_2 = R$ and $A_1 = A_2$: We postulate that, in this sub-game, consumers do not search and both firms charge $p^i = \frac{E[A_i]+\bar{\gamma}}{2}$. First, since both firms charge the same price, and the consumers receive the first quote from the firm that matches their taste, consumers have no incentive to search for a second quote. In other words, there is no profitable deviation for the consumers. Second, because the consumers do not search for a second quote, neither firm can steal consumers from the other through reduced prices. Then, the pricing problem of firm i boils down to:

$$\max_{p_i > 0} p_i \left(1 + \frac{A_i - p_i + \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}}\right) \tag{A22}$$

Hence, changing prices does not increase profits, since $p^i = \frac{E[A_i]+\bar{\gamma}}{2}$ is already the price that equates marginal revenue to marginal cost for firm i. Thus, the postulated strategies constitute a Nash Equilibrium of the pricing-search sub-game.

If $x_2 = L$ and $A_1 = A_2$: We postulate that, in this sub-game, consumers do not search and both firms charge $p^i = \frac{E[A_i]+\bar{\gamma}}{2} - \frac{\delta}{4}$ for δ small enough and $p^i = \frac{E[A_i]+\bar{\gamma}}{2}$ otherwise. First, since both firms charge the same price, and $A_1 = A_2$, consumers have no incentive to search for a second quote. Second, because the consumers do not search for a second quote, neither firm can steal consumers from the other through reduced prices. Then, the pricing problem of a monopolist boils down to:

$$\max_{p_i > 0} \frac{1}{2} p_i \left(\max\{1 + \frac{A_i - p_i + \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}}, 0\} + \max\{\frac{A_i - p_i + \underline{\gamma} - \delta}{\bar{\gamma} - \underline{\gamma}}, 0\}\right) \tag{A23}$$

The pricing problem looks different from (A22) because now, if the price is sufficiently small, firm i can serve consumers whose tastes do not exactly match firm i's product, i.e. consumers located in R.[37]

The problem may be non-convex around the solution, due to the presence of two separate markets: it may be optimal for the firm to set a price where the demand from location R equals 0. This becomes more likely as δ grows. First, assume that the firm serves both markets in the optimal solution. Then, the price that solves

$$\max_{p_i > 0} \frac{1}{2} p_i \left(1 + \frac{A_i - p_i + \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}} + \frac{A_i - p_i + \underline{\gamma} - \delta}{\bar{\gamma} - \underline{\gamma}}\right), \tag{A24}$$

or, $p_i^* = \frac{E[A_i]+\bar{\gamma}}{2} - \frac{\delta}{4}$, also solves (A23). If, on the other hand, $A_i - p_i^* + \underline{\gamma} - \delta < 0$, then the firm will charge $p_i^{**} = \frac{E[A_i]+\bar{\gamma}}{2}$ and only serve consumers located at L. Hence, one of these prices equates

[37] The $\frac{1}{2}$ term in the beginning signifies the fact that only half of the consumers in either location receive their quote from firm i.

marginal revenue to marginal cost for firm i. Thus, the postulated strategies constitute a Nash Equilibrium of the pricing-search sub-game.

If $A_1 \neq A_2$: We postulate that, in this sub-game, consumers do not search, and both firms charge $p^i = \frac{E[A_i]+\bar{\gamma}}{2} - \frac{\delta}{4}$ for δ small enough and $p^i = \frac{E[A_i]+\bar{\gamma}}{2}$ otherwise. The strategies here are identical to the previous sub-game, and proving that they constitute a Nash Equilibrium follows the same steps. The only difference in this case is that consumer j buys from the firm that has a larger $E[A_i] - |x_i - \chi_j|$.[38] □

Proof of Proposition 5. We restrict attention to the case where monopolists serve both markets, i.e., where δ is sufficiently small. This is also the interesting case where co-location leads to reduced prices for consumers.

The PBE is defined by:

1. $x^2 = L$

2. $$a^i(x_i = R, \theta) = \begin{cases} P_i, \text{ if } N_i = 0, P_{-i} = 0, P_i = \Pi \\ N_{-i}, \text{if above condition fails and } N_{-i} = -\beta \\ P_i, \text{ if above conditions fail and } P_i = \Pi \\ \varnothing, \text{otherwise} \end{cases}$$
and
$$a^i(x_i = L, \theta) = \begin{cases} P_i, \text{if } P_i = \Pi \\ N_{-i}, \text{if above condition fails and } N_{-i} = -\beta \\ \varnothing, \text{otherwise} \end{cases}$$

3. $\mathscr{F}$ is as described in Table 1.

4. $s^j(.) = \text{not}, \forall \theta, a_1, a_2, x_2$

5. $$p^i(x_i = R, .) = \begin{cases} \frac{E[A_i]+\bar{\gamma}}{2}, \text{if } A_1 = A_2 \\ \frac{E[A_i]+\bar{\gamma}}{2} - \frac{\delta}{4}, \text{otherwise} \end{cases}$$
and
$$p^i(x_i = L, .) = \left\{ \frac{E[A_i]+\bar{\gamma}}{2} - \frac{\delta}{4} \right.$$

[38] The assumption that consumers receive the first quote from the firm with larger $E[A_i] - |x_i - \chi_j|$ greatly simplifies characterizing the equilibrium. Otherwise, some consumers would be better off searching for a second quote even under identical prices, which would create incentives for firms to undercut each other's prices.

6. Let $B_{ij} = A_i - |\chi_j - x_i| - p_i$

- if $\gamma_j < \max_{i \in \mathscr{S}_j} E[B_{ij}]$ then $g^j(x_2, a_1, a_2) = \varnothing$
- if above condition fails and $\arg\max_{i \in \mathscr{S}_j} E[B_{ij}]$ is unique, then $g^j(x_2, a_1, a_2) = \arg\max_i E[B_{ij}]$
- otherwise $g^j(x_2 = L, a_1, a_2) = \begin{cases} 1, \text{w.p. } 0.5 \\ 2, \text{w.p. } 0.5 \end{cases}$ and $g^j(x_2 = R, a_1, a_2) = \begin{cases} 1, \text{ if } \chi_j = L \\ 2, \text{ if } \chi_j = R \end{cases}$

where $\mathscr{S}_j = \{1, 2\}$ if s^j=search and $\mathscr{S}_j = \{i\}$ otherwise, where i denotes the product whose free quote is received by consumer j.

We prove that $\{x^2, a^1, a^2, \mathscr{F}, g^j, s^j, p^1, p^2\}$ is a PBE when

(i) $\beta > (1 - \sigma_\Pi)\Pi + \delta$ (A25a)

(ii) $(\bar{\gamma} + \sigma_\beta\beta)(\bar{\gamma} + \sigma_\beta\beta - 2(1 - \sigma_\Pi)\Pi - 2\delta) \geq ((1 - \sigma_\Pi)\Pi - 0.5\delta)((1 - \sigma_\Pi)\Pi + \delta)$ (A25b)

(iii) $\Pi > (1 - \rho)(1 - \sigma_\beta)\beta$ (A25c)

(iv)
$$\begin{aligned}
&\big(\sigma_\Pi^2 + \rho\sigma_\Pi(1 - \sigma_\Pi)\big)(\bar{\gamma} + (1 - \sigma_\Pi)\Pi)^2 \\
&+ 2\rho(1 - \sigma_\Pi)(1 - \rho)\sigma_\beta(\bar{\gamma} + (1 - \sigma_\Pi)\Pi - (1 - \sigma_\beta)\beta)^2 \\
&+ 2\sigma_\Pi(1 - \sigma_\Pi)(1 - \rho)(1 - \sigma_\beta)(\bar{\gamma} + (1 - \sigma_\Pi)\Pi + \sigma_\beta\beta)^2 \\
&+ (1 - \sigma_\Pi)(1 - \sigma_\Pi + \rho\sigma_\Pi)(\sigma_\beta^2 + \rho\sigma_\beta(1 - \sigma_\beta))(\bar{\gamma} - \sigma_\Pi\Pi - (1 - \sigma_\beta)\beta)^2 \\
&+ (1 - \sigma_\Pi)(1 - \sigma_\beta)(1 - \sigma_\Pi + \rho\sigma_\Pi)(1 + \sigma_\beta - \rho\sigma_\beta)(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - 0.5\delta^2 \\
&\geq \big(2\sigma_\beta(1 - \sigma_\beta)\sigma_\Pi + (1 - \sigma_\beta)^2(1 - \sigma_\Pi)^2\big)(\bar{\gamma} + \sigma_\beta\beta)^2 \\
&+ (1 - \sigma_\beta)^2\sigma_\Pi(2 - \sigma_\Pi)(\bar{\gamma} + \sigma_\beta\beta + (1 - \sigma_\Pi)\Pi)^2 + \sigma_\beta^2(\bar{\gamma} - (1 - \sigma_\beta)\beta)^2 \\
&+ (1 - \sigma_\beta)(1 - \sigma_\Pi)(1 + \sigma_\beta - \sigma_\Pi + \sigma_\Pi\sigma_\beta)(\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi)^2 \\
&- 0.5(1 - \sigma_\beta)(\sigma_\beta + (1 - \sigma_\beta)\sigma_\Pi(1 - \sigma_\Pi))\delta^2
\end{aligned} \tag{A25d}$$

First, g^j, by definition, maximizes $E[U_{ij}]$ given $\mathscr{F}$ and s_j. Second, $\mathscr{F}$ is consistent with the advertising strategies of the firm types followed in equilibrium.

Third, we discuss the optimality of $a^i(.)$ given $\mathscr{F}$. Notice that positive advertising would be prioritized against 'weak opponents' after both location choices for the same reasons as described in the Proof of Proposition 2.

Locating Apart: If the opponent is strong, then, for negative advertising to be prioritized in the advertising equilibrium, negative advertising should be effective enough to steal consumers if the opponent runs positive advertising. Since consumers don't search in equilibrium and prices are symmetric, firm i would steal consumers by guaranteeing $E[A_i - A_{-i}] > \delta$:

$$\begin{aligned} E[A_i - A_{-i}|a_i = N_i, a_{-i} = P_i] > \delta \\ \Leftrightarrow \beta - (1-\sigma_\Pi)\Pi > \delta, \end{aligned} \tag{A26}$$

which is equivalent to condition (A25a) above. Second, for negative advertising to be prioritized against strong opponents in the unique advertising equilibrium, the revenues from stolen consumers should be sufficient to make up for the lost revenues in own location ($R_i(a_i = N_i, a_{-i} = P_i) \geq R_i(a_i = P_i, a_{-i} = P_i)$):

$$\left(\frac{\sigma_\beta\beta + \overline{\gamma}}{2} - \frac{\delta}{4}\right)\left(2 - \Gamma\big(\frac{\sigma_\beta\beta + \overline{\gamma}}{2} - \frac{\delta}{4} - \sigma_\beta\beta\big) - \Gamma\big(\frac{\sigma_\beta\beta + \overline{\gamma}}{2} - \frac{\delta}{4} + \delta - \sigma_\beta\beta\big)\right) \geq$$
$$\left(\frac{(1-\sigma_\Pi)\Pi + \sigma_\beta\beta + \overline{\gamma}}{2}\right)\left(1 - \Gamma\big(\frac{(1-\sigma_\Pi)\Pi + \sigma_\beta\beta + \overline{\gamma}}{2} - \sigma_\beta\beta - (1-\sigma_\Pi)\Pi\big)\right),$$

which can be simplified as

$$(\bar{\gamma} + \sigma_\beta\beta)(\bar{\gamma} + \sigma_\beta\beta - 2(1-\sigma_\Pi)\Pi - 2\delta) \geq ((1-\sigma_\Pi)\Pi - 0.5\delta)((1-\sigma_\Pi)\Pi + \delta),$$

which is equivalent to condition (A25b) above. Since the game is symmetric, conditions (A25a) and (A25b) together imply prioritizing negative advertising against strong opponents is strictly dominant. See that (A25b) is harder to satisfy than (A1b) because

$$\begin{aligned} (\bar{\gamma} + \sigma_\beta\beta - 2(1-\sigma_\Pi)\Pi - 2\delta) > ((1-\sigma_\Pi)\Pi - 0.5\delta) \\ \Leftrightarrow \ \bar{\gamma} + \sigma_\beta\beta > 3(1-\sigma_\Pi)\sigma_\Pi + \frac{3\delta}{2}, \end{aligned}$$

is harder to satisfy than

$$\bar{\gamma} + \sigma_\beta\beta > (1-\sigma_\Pi)\Pi + \delta.$$

Hence, negative advertising under locating apart is less likely when pricing is introduced to the model. Because this is a necessary condition for the entrant to co-locate, co-location is also less likely when pricing is introduced.[39]

Co-Locating: If the opponent is strong, for positive advertising to be prioritized in the unique advertising equilibrium, positive advertising should be effective enough to steal consumers if the opponent runs negative advertising. Since consumers don't search in equilibrium and prices are

[39] Condition (A25d) also changes once pricing is introduced. However, condition (A25b) has precedence over condition (A25d) because (A25d) assumes (A25b) is satisfied.

symmetric, firm i would steal consumers by guaranteeing $E[A_i - A_{-i}] > 0$:

$$\begin{aligned} E[A_i - A_{-i} | a_i = P_i, a_{-i} = N_i] > 0 \\ \Pi > (1-\rho)(1-\sigma_\beta)\beta, \end{aligned} \tag{A27}$$

which is equivalent to condition (A25c) above.

Fourth, to prove the optimality of x^2, consider the potential outcomes following each location choice, summarized in Table A14.

θ	a_1	a_2	p_2 $(\frac{1}{2}*)$	R_2 $(\frac{1}{4(\bar{\gamma}-\underline{\gamma})}*)$
$\{\Pi, \Pi, , \}$	P_1	P_2	$\bar{\gamma} + (1-\sigma_\Pi)\Pi - \frac{\delta}{2}$	$(\bar{\gamma} + (1-\sigma_\Pi)\Pi)^2 - \frac{\delta^2}{4}$
$\{\Pi, 0, -\beta, \}$	P_1	N_1	$\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta - \frac{\delta}{2}$	0
$\{0, \Pi, , -\beta\}$	N_2	P_2	$\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta - \frac{\delta}{2}$	$2\big((\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta)^2 - \frac{\delta^2}{4}\big)$
$\{\Pi, 0, 0, \}$	P_1	$\varnothing$	$\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta - \frac{\delta}{2}$	0
$\{0, \Pi, , 0\}$	$\varnothing$	P_2	$\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta - \frac{\delta}{2}$	$2\big((\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$
$\{0, 0, -\beta, -\beta\}$	N_2	N_1	$\bar{\gamma} - \sigma_\Pi\Pi - (1-\sigma_\beta)\beta - \frac{\delta}{2}$	$(\bar{\gamma} - \sigma_\Pi\Pi - (1-\sigma_\beta)\beta)^2 - \frac{\delta^2}{4}$
$\{0, 0, 0, -\beta\}$	N_2	$\varnothing$	$\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta - \frac{\delta}{2}$	0
$\{0, 0, -\beta, 0\}$	$\varnothing$	N_1	$\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta - \frac{\delta}{2}$	$2\big((\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$
$\{0, 0, 0, 0\}$	$\varnothing$	$\varnothing$	$\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta - \frac{\delta}{2}$	$(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}$

Table A12: Co-Location

θ	a_1	a_2	p_2 $(\frac{1}{2}*)$	R_2 $(\frac{1}{4(\bar{\gamma}-\underline{\gamma})}*)$
$\{, , -\beta, -\beta\}$	N_2	N_1	$\bar{\gamma} - (1-\sigma_\beta)\beta$	$(\bar{\gamma} - (1-\sigma_\beta\beta)^2$
$\{, \Pi, 0, -\beta\}$	N_2	P_2	$\bar{\gamma} + \sigma_\beta\beta - \frac{\delta}{2}$	0
$\{\Pi, , -\beta, 0\}$	P_1	N_1	$\bar{\gamma} + \sigma_\beta\beta - \frac{\delta}{2}$	$2\big((\bar{\gamma} + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$
$\{, 0, 0, -\beta\}$	N_2	$\varnothing$	$\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$	0
$\{0, , -\beta, 0\}$	$\varnothing$	N_1	$\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$	$2\big(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$
$\{\Pi, \Pi, 0, 0\}$	P_1	P_2	$\bar{\gamma} + \sigma_\beta\beta + (1-\sigma_\Pi)\Pi$	$(\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2$
$\{\Pi, 0, 0, 0\}$	P_1	$\varnothing$	$\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$	0
$\{0, \Pi, 0, 0\}$	$\varnothing$	P_2	$\bar{\gamma} + \sigma_\beta\beta + (1-\sigma_\Pi)\Pi - \frac{\delta}{2}$	$2\big((\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2 - \frac{\delta^2}{4}\big)$
$\{0, 0, 0, 0\}$	$\varnothing$	$\varnothing$	$\bar{\gamma} + \sigma_\beta\beta - \sigma_\Pi\Pi - \frac{\delta}{2}$	$(\bar{\gamma} - \sigma_\Pi\Pi + \sigma_\beta\beta)^2$

Table A13: Locating Apart

Table A14: Revenues for the Entrant with Negative Advertising Allowed, p_2 and R_2 refer to the price and revenues for the entrant for each outcome. The terms in the paranthesis are common multipliers for all values in the columns. $\theta = \{P_1, P_2, N_1, N_2\}$ denotes the realizations for attributes. If the associated entry in θ is unspecified, that means a_i, p_i, and D_i do not depend on the value of that entry. See Table 1 for $E[A_1]$ and $E[A_2]$ associated with each outcome. Refer to Table A6 for the associated probabilities for each scenario.

With some algebra, we can simplify the expected payoff after each location choice to

$$\begin{aligned} E[R_2 | x_2 = L] = \frac{1}{4(\bar{\gamma} - \underline{\gamma})} \Big[&\big(\sigma_\Pi^2 + \rho\sigma_\Pi(1-\sigma_\Pi)\big)(\bar{\gamma} + (1-\sigma_\Pi)\Pi)^2 \\ &+ 2\rho(1-\sigma_\Pi)(1-\rho)\sigma_\beta(\bar{\gamma} + (1-\sigma_\Pi)\Pi - (1-\sigma_\beta)\beta)^2 \\ &+ 2\sigma_\Pi(1-\sigma_\Pi)(1-\rho)(1-\sigma_\beta)(\bar{\gamma} + (1-\sigma_\Pi)\Pi + \sigma_\beta\beta)^2 \\ &+ (1-\sigma_\Pi)(1-\sigma_\Pi + \rho\sigma_\Pi)(\sigma_\beta^2 + \rho\sigma_\beta(1-\sigma_\beta))(\bar{\gamma} - \sigma_\Pi\Pi - (1-\sigma_\beta)\beta)^2 \end{aligned} \tag{A28}$$

$$+ (1-\sigma_\Pi)(1-\sigma_\beta)(1-\sigma_\Pi+\rho\sigma_\Pi)(1+\sigma_\beta-\rho\sigma_\beta)(\bar{\gamma}-\sigma_\Pi\Pi+\sigma_\beta\beta)^2 - 0.5\delta^2\Big]$$

$$E[R_2|x_2=R] = \frac{1}{4(\bar{\gamma}-\underline{\gamma})}\Big[\big(2\sigma_\beta(1-\sigma_\beta)\sigma_\Pi + (1-\sigma_\beta)^2(1-\sigma_\Pi)^2\big)(\bar{\gamma}+\sigma_\beta\beta)^2 \quad \text{(A29)}$$

$$+ (1-\sigma_\beta)^2\sigma_\Pi(2-\sigma_\Pi)(\bar{\gamma}+\sigma_\beta\beta+(1-\sigma_\Pi)\Pi)^2 + \sigma_\beta^2(\bar{\gamma}-(1-\sigma_\beta)\beta)^2$$

$$+ (1-\sigma_\beta)(1-\sigma_\Pi)(1+\sigma_\beta-\sigma_\Pi+\sigma_\Pi\sigma_\beta)(\bar{\gamma}+\sigma_\beta\beta-\sigma_\Pi\Pi)^2$$

$$- 0.5(1-\sigma_\beta)(\sigma_\beta+(1-\sigma_\beta)\sigma_\Pi(1-\sigma_\Pi))\delta^2\Big].$$

Hence, $E[R_2|x_2 = L] \geq E[R_2|x_2 = R]$ becomes equivalent to condition (A25d) above.

Last, the fact that consumers' search decisions and firms' pricing decisions constitute a Nash equilibrium of the pricing-search subgame has been established in Lemma 8.

To sum up, once conditions (A25a)-(A25d) are satisfied, there exists a PBE as defined in $1-6$. □

Proof of Proposition 6. First, consider incentives to run negative advertising against weak opponents where the entrant locates apart. In product competition, the necessary condition for negative advertising to be prioritized was $E[A_i - A_{-i}|a_i = N] > \delta \geq E[A_i - A_{-i}|a_i = P]$, i.e., only negative advertising allows stealing consumers. In political competition, negative advertising may be utilized even when it doesn't lead to stolen consumers. As long as the opponent loses sufficiently many consumers, negative advertising can be utilized. In other words, the necessary condition is $E[A_i - A_{-i}|a_i = N] \geq E[A_i - A_{-i}|a_i = P]$, which is weaker than the condition above. In the scenario where the entrant co-locates, there would be no change in the incentives and positive advertising would be prioritized against weak opponents in both political and product competition.

Second, consider incentives to run negative advertising against strong opponents. In product competition, there are two necessary conditions for negative advertising to be prioritized: (1) negative advertising allows stealing consumers when the competitor runs positive advertising and (2) the number of stolen consumers is sufficiently large. In political competition, the first condition is sufficient by itself, because a decline in total number of votes is not problematic as long as the opponent loses more votes. Hence, negative advertising is more likely against strong opponents under both co-location and locating apart.

To sum up, under any parameter set where negative advertising is prioritized in product competition, negative advertising is also prioritized in political competition. □

Proof of Proposition 7. (i) In the benchmark where $\eta = 0$, the game is symmetric between the firms. Hence, the expected winning probability is 0.5 following both co-location and locating apart under political competition. This result is independent of which equilibrium is played in the advertising subgames, because all advertising equilibria are symmetric.

(ii) In political competition, the presence of $\eta > 0$ implies that co-location leads to a strictly smaller winning probability for the entrant. This is because the winning probability is discontinuous in vote difference, hence it is discontinuous at $\eta = 0$. When a mass $\eta > 0$ prefers the incumbent, then any symmetric advertising choice leads to incumbent winning the race with probability 1 (instead of 0.5), reducing the ex-ante winning probability of the entrant following co-location strictly below 0.5.

If the entrant candidate locates apart, however, there exists a small enough $\eta > 0$ such that the winning probability of the entrant is still 0.5. Under locating apart, η only matters when the advertising outcomes favor the entrant more than the incumbent to an extent where entrant can steal the regular voters but not the voters which have the incumbency bias (measure $1 - \eta$). In that case, the entrant would win with probability 1 as long as $\eta < 0.5$. For other advertising outcomes, voters in L would vote for the incumbent independent of the incumbency advantage. Hence, for $\eta < 0.5$, the winning probability of the entrant is still 0.5 following locating apart. In short, for $\eta > 0$ small enough, the entrant candidate would locate apart.

The same reasoning does not work in product competition because profits/demand are continuous in η. The presence of η can only reduce the expected demand for the entrant firm by an amount proportional to η. Hence, when η is small enough, its impact on the expected payoffs is negligible. For any situation where the entrant strictly prefers to co-locate, there exists an η small enough so that the entrant still strictly prefers to co-locate. □

Proof of Proposition A.1. Part (i) The proof follows the steps of the of Proof for Proposition 2 almost line by line. The steps for the optimality of purchase and advertising decisions, and how consumer beliefs $\mathscr{F}$ satisfy the Bayes rule given the advertising decisions is identical to the proof for Proposition 2. The only difference now is that location choice is not made in isolation by the entrant, but decided simultaneously within a game for two firms. Hence, the updated equilibrium condition would state that location decisions x^1 and x^2 constitute a Nash equilibrium. For the parameter set where the entrant decides to co-locate, the expected payoff following co-location should be larger than the expected payoff following locating apart, conditional on the location of the incumbent. Then, the Nash equilibria of the location decisions game (conditional on g^j, a^1 and a^2) would be $\{x^1, x^2\} = \{L, L\}$ and $\{x^1, x^2\} = \{R, R\}$. If $\{x^1, x^2\} = \{L, R\}$ or $\{x^1, x^2\} = \{R, L\}$, then both firms would have a profitable deviation to the other location. The observable equilibrium outcomes are identical to those of the entrant-incumbent game, up to a symmetric change in where firms are located.

Part (ii) Similarly, for the parameter set where the entrant decides to locate apart, the expected payoff following locating apart should be larger than the expected payoff following co-location, conditional on the location of the incumbent. Then, the Nash equilibria of the location decision game (conditional on g^j, a^1 and a^2) would be $\{x^1, x^2\} = \{L, R\}$ and $\{x^1, x^2\} = \{R, L\}$. If $\{x^1, x^2\} = \{L, L\}$ or $\{x^1, x^2\} = \{R, R\}$, then both firms would have a profitable deviation to the other location. The observable equilibrium outcomes are identical to those of the entrant-incumbent game, up to a symmetric change in where firms are located. $\square$